Materials of Conferences

ON MULTICRITERIA MODELING PROBLEM

Matusov J.

*Mechanical Engineering Research Institute,
Russian Academy of Sciences, Moscow,
e-mail: matusoff.l@yandex.ru*

The engineering optimization and identification problems are essentially multicriteria. The multicriteria method of identification of a mathematical models are considered in our work.

The Formulation of Multicriteria Optimization Problems

Let us consider an object whose operation is described by a mathematical model (system of equations) or whose performance criteria may be directly calculated. We assume that the system depends on *r* design variables $\alpha_1,...,\alpha_r$ representing a point $\alpha = (\alpha_1,...,\alpha_r)$ of an *r*-dimensional space. In the general case one has to take into account design variable, functional, and criteria constraints [1] .There exist particular performance criteria, such as productivity, materials consumption, and efficiency. It is desired that, with other things being equal, these criteria, denoted by $\Phi_\nu(\alpha)$, $\nu = 1,...,k$ would have the extremal values. For simplicity, we assume that $\Phi_\nu(\alpha)$, are to be minimized. In order to avoid situations in which the expert regards the values of some criteria as unacceptable, we introduce criteria constraints $\Phi_\nu(\alpha) \le \Phi_\nu^{**}$, $\nu = 1,...,k$, where Φ_ν^{**} is the worst value of criterion to which the expert may agree. All these constraints) define the feasible solution set *D* [1].

Definition. A point $\alpha^0 \in D$, is called the Pareto optimal point if there exists no point $\alpha \in D$ such that $\Phi_\nu(\alpha) \le \Phi_\nu(\alpha^0)$ for all $\nu = 1,..., k$ and $\Phi_{\nu_0}(\alpha) < \Phi_{\nu_0}(\alpha^0)$ for at least one $\nu_0 \in \{1,\dots,k\}$.

A set $P \subset D$ is called a Pareto optimal set if it consists of Pareto optimal points. When solving the problem, one has to determine design variable vector $\alpha^0 \in P$, which is the most preferable among the vectors belonging to set *P*.

The Pareto optimal set plays an important role in vector optimization problems because it can be analyzed more easily than the feasible solution set and because the optimal vector always belongs to the Pareto set, irrespective of the system of preferences used by the expert for comparing vectors belonging to the feasible solution set. Solving the specified problems was made possible owing to the PSI method [1].

Multicriteria identification (modeling). Adequacy of mathematical models

In the most common usage, the term “identification” means construction of the mathematical model of a system and determination of the parameters α_j (design variables) of the model by using the information about the system response to known external disturbances. Very often, when solving identification problems, the researcher has no information about the limits α_j^* and α_j^{**} for many of the variables. As a rule these applied identification problems have been treated as single-criterion problems. In the majority of conventional problems, the system is tacitly assumed to be in full agreement with its mathematical model. However, for complex engineering systems we generally cannot assert a sufficient correspondence between the model and the object. This does not permit us to use a single criterion to evaluate the adequacy. In multicriteria identification problems there is no necessity of artificially introducing a single criterion to the detriment of the physical essence of the problem. When constructing a mathematical model one first defines the class and structure of the model operator, that is, the law according to which disturbances (input variables) are transformed into the system response (output processes). This is called structural identification. For mechanical systems structural identification means determining the type and number of equations constituting the mathematical model of the system. Parametric identification is reduced to finding numerical values of the equation coefficients, based on the realization of the input and output processes. In doing so, frequency responses, transfer functions, and unit step functions are often used. A number of problems require preliminary experimental determination of the basic characteristics of a mechanical system (e.g., the frequencies, shapes, and decrements of natural oscillations). When solving optimization problems, we have used the concept of performance criterion. In identification problems we will deal with particular adequacy (proximity) criteria. By adequacy (proximity) criteria we mean the discrepancies between the experimental and computed data, the latter being determined on the basis of the mathematical model. In all basic units of the structure under study we experimentally measure the values of the characteristic quantities of interest (e.g., displacements, velocities, accelerations, etc.). At the same time we calculate the corresponding quantities by using the mathematical model. As a result, particular adequacy (proximity) criteria are formed as functions of the difference between the experimental and computed data. Thus we arrive at a multicriteria problem. The multicriteria consideration makes it possible to extend the application area of the identification theory substantially.

Parameter Space Investigation Method in Problems of Multicriteria Identification

We denote by $\Phi_\nu^c(\alpha)$, $\nu = \overline{1,k}$ the indices (criteria) resulting from the analysis of the mathematical model that describes a physical system, where $\alpha = (\alpha_1, \ldots, \alpha_r)$ is the vector of the parameters of the model. Let $\Phi_\nu^{\exp}$ be the experimental value of the ν th criterion measured directly on the prototype. Suppose there exists a mathematical model or a hierarchical set of models describing the system behaviour. Let $\Phi = \left(\left\|\Phi_1^c - \Phi_1^{\exp}\right\|, \ldots, \left\|\Phi_k^c - \Phi_k^{\exp}\right\|\right)$, where $\|\cdot\|$ is a particular adequacy (closeness, proximity) criterion. This criterion, as has already been mentioned, is a function of the difference (error) $\Phi_\nu^c - \Phi_\nu^{\exp}$. Very often it is given by $\left(\Phi_\nu^c - \Phi_\nu^{\exp}\right)^2$ or $\left|\Phi_\nu^c - \Phi_\nu^{\exp}\right|$. If the experimental values $\Phi_\nu^{\exp}$, $\nu = \overline{1,k}$ are measured with considerable error, then the quantity $\Phi_\nu^{\exp}$ can be treated as a random variable. If this random variable is normally distributed, the corresponding adequacy criterion is expressed by $M\left\{\left\|\Phi_\nu^c - \Phi_\nu^{\exp}\right\|\right\}$, where $M\{\|\cdot\|\}$ denotes the mathematical expectation of the random variable $\|\cdot\|$. For other distribution functions, more complicated methods of estimation are used, for example, the maximum likelihood method. We formulate the following problem by comparing the experimental and calculation data, determining to what extent the model corresponds to the physical system, and finding the variables of the model. In other words, it is necessary to find the vectors α^i satisfying design variable, functional, and criteria constraints design variable, functional, and criteria constraints and, in addition, the inequalities $\left\|\Phi_\nu^c\left(\alpha^i\right) - \Phi_\nu^{\exp}\right\| \le \Phi_\nu^{**}$.

All these conditions defines the feasible solution set D_α. Here, Φ_ν^{**} are criteria constraints that are determined in the dialogue between the researcher and a computer. To a considerable extent, these constraints depend on the accuracy of the experiment and the physical sense of the criteria.

The Search for the Identified Solutions

The formulation and solution of the identification problem are based on the parameter space investigation method. In accordance with the algorithm given above, we specify the values Φ_ν^{**} and find vectors meeting above meanshioned conditions. The vectors α_{id}^i belonging to the feasible solution set D_α will be called adequate vectors. The vectors α_{id}^i that belong to the set of adequate vectors and have been chosen by using a special decision making rule will be called identified vectors.

The role of the decision making rule is often played by nonformal analysis of the set of adequate vectors. If this analysis separates several equally acceptable vectors α_{id}^i, the solution of the identification problem is nonunique.

The identified vectors α_{id}^i form the identification domain $D_{id} = \bigcup_i \alpha_{id}^i$. Sometimes, by carrying out additional physical experiments, revising constraints Φ_ν^{**}, etc., one can reduce the domain D_{id} and even achieve the result that this domain contains only one vector. Unfortunately, this is far from usual. Nonunique restoration of variables is a recompense for the discrepancy between the physical object and its mathematical model, incompleteness of physical experiments, etc. If a mathematical model is sufficiently good (i.e., it correctly describes the behaviour of the physical system), then multicriteria parametric identification leads to a nonempty set D_α. The most important factors that can lead to an empty D_α are imperfection of the mathematical model and lack of information about the domain in which the desired solutions should be searched for. The search for the set D_α is very important, even in the case where the results are not promising. It enables the researcher to judge the mathematical model objectively (not only intuitively), to analyze its advantages and drawbacks on the basis of all proximity criteria, and to correct the problem formulation. Thus, multicriteria identification includes the determination and nonformal analysis of the feasible solution set D_α with regard to all basic proximity criteria, as well as finding identified solutions α_{id}^i belonging to this set. Multicriteria identification is often the only way to evaluate the quality of the mathematical model and, hence, to optimize this model. The algorithm is successfully used in practice. Below we discuss some important problems that are solved by using this algorithm.

References

1. Statnikov R.B., and J.B. Matusov, Multicriteria Analysis in Engineering, Dordrecht/Boston/London: Kluwer Academic Publishers, 2002.

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