A VERY INEXPENSIVE SCHEME ON RFEM TO USE IN CFD AND OTHER PROBLEMS

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Note, Reference [1] should be read before reading this paper. In this paper I am going to use the Reduced Finite Element Method (RFEM), see [1] with a constant amount of Element Degree Limiter, Field Variable Limiter (EDL, FVL) coefficient to obtain an inexpensive scheme of non-oscillatory, for this purpose I used third degree Lagrangian elements on full upwind difference scheme (FUDS) and its result was very successful.

Keywords: Reduced Finite Element Method, Non-oscillatory, Full upwind difference scheme, Third-order scheme.

The non-oscillatory shape functions [1] is written as follows:

$$\begin{aligned} N_0^{EDL,FVL} &= (1-\delta_i)N_0^{(p)} + \delta_i N_0^{(ref)} \\ N_j^{EDL,FVL} &= (1-\delta_i)N_j^{(p)} + \delta_i N_j^{(ref)} \\ N_L^{EDL,FVL} &= (1-\delta_i)N_L^{(p)} + \delta_i N_L^{(ref)} \end{aligned} \qquad (1)$$

Where δ_i is Element Degree Limiter, Field Variable Limiter (EDL, FVL), $N^{(p)}$ is p-order shape functions (in this paper degree of p is 3) and $N_0^{(ref)}$, $N_j^{(ref)}$ and $N_L^{(ref)}$ are reference functions I used equation (40) of [1] as reference function that is

$$N_0^{(ref)} = N_0^{(1)}, \quad N_j^{(ref)} = 0, \quad N_L^{(ref)} = N_L^{(1)} \qquad (2)$$

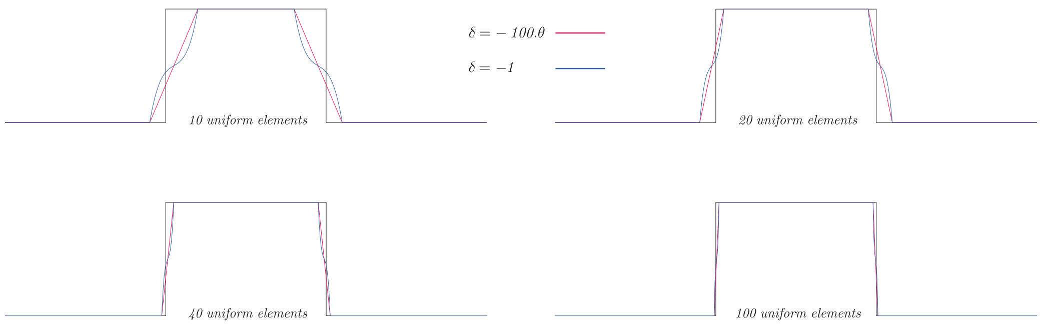

Fig. 1. 1D advective equation $u_t + u_x = 0$

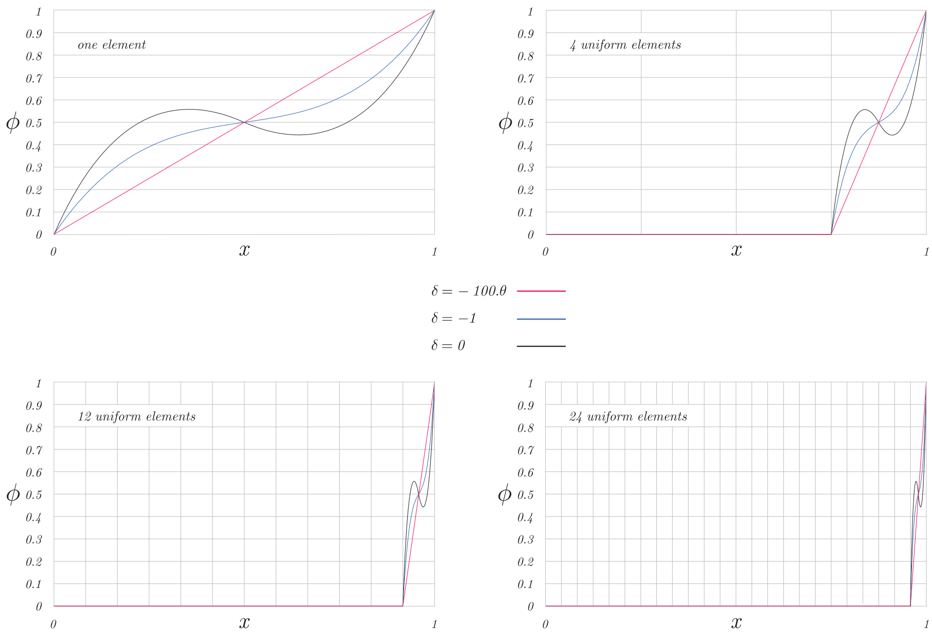

Fig. 2. 1D convection-diffusion equation $a\phi_x - k\phi_{xx} = 0$

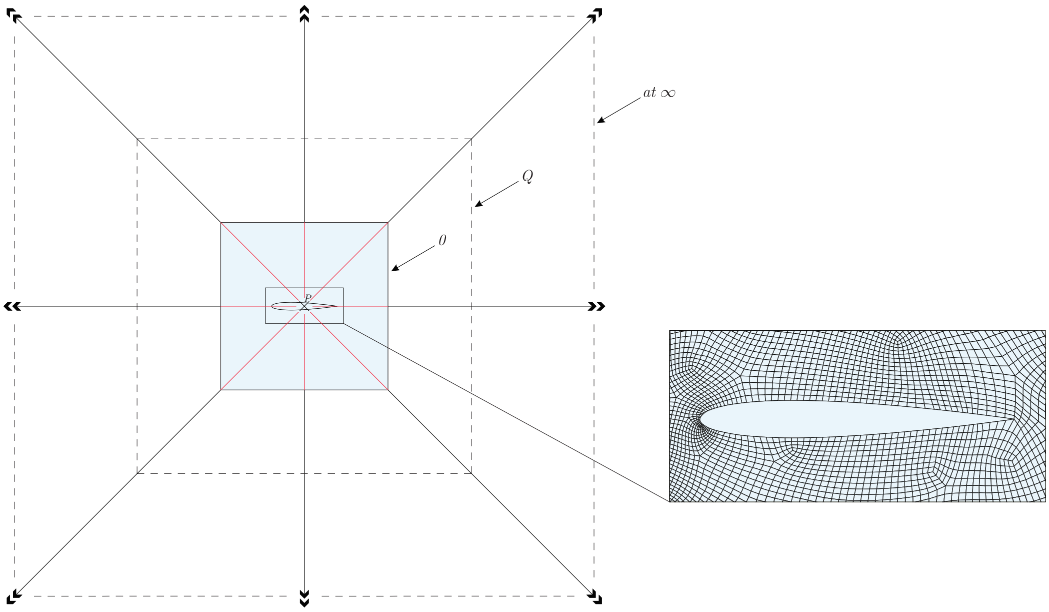

Fig. 3. Grid for flow over a NACA 0012 airfoil

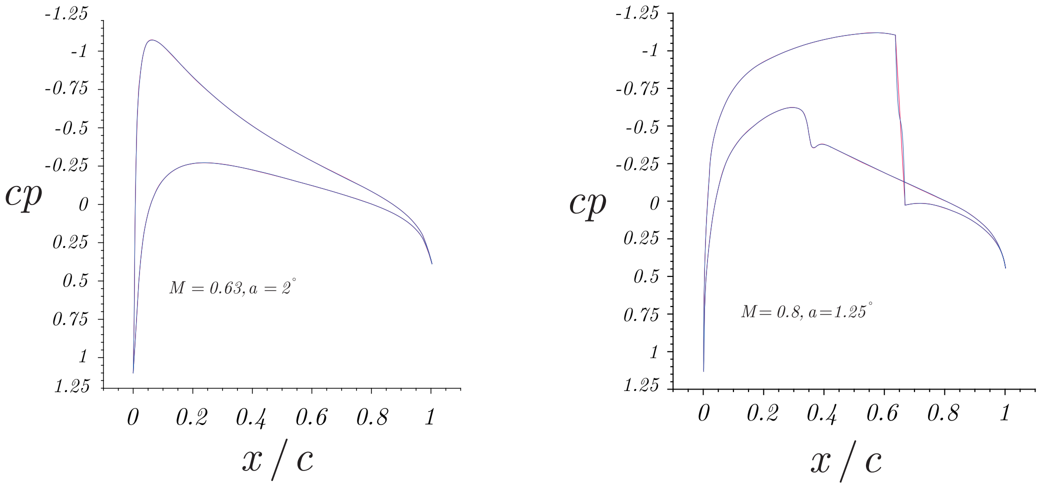

Fig. 4. Pressure distribution on surface of NACA 0012 airfoil, steady-state

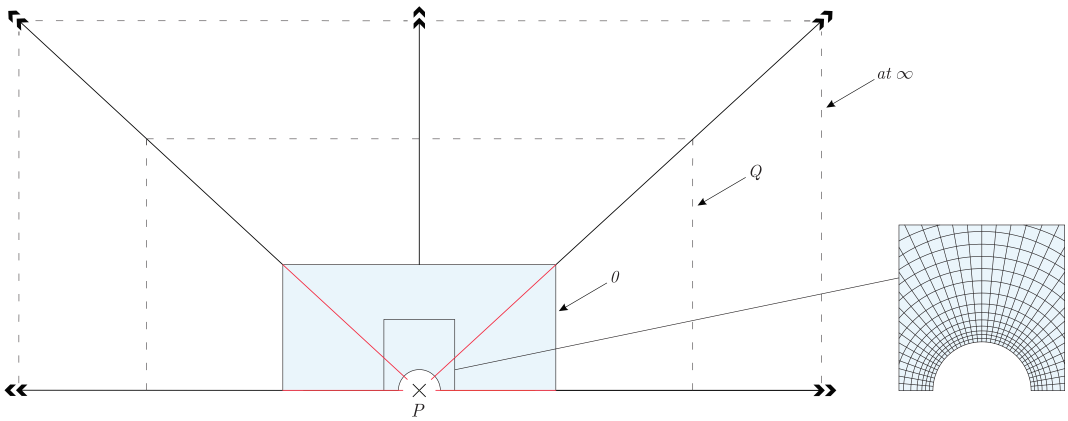

Fig. 5. Grid for flow around a cylinder.

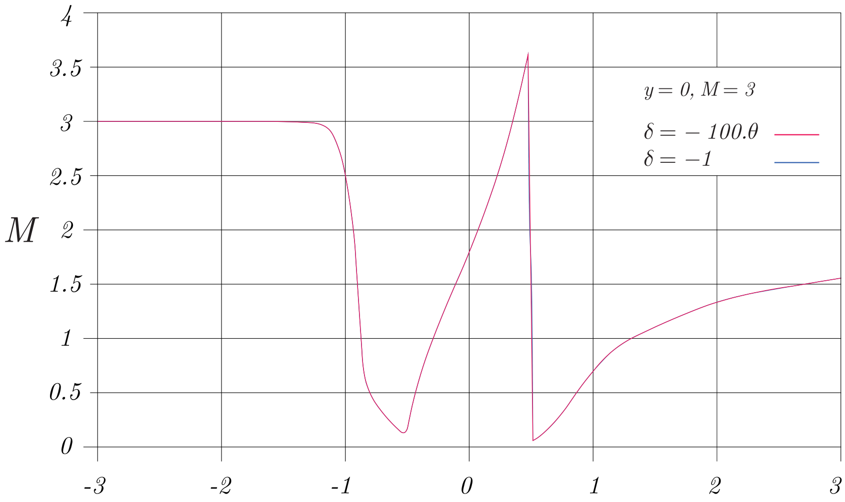

Fig. 6. Flow around a cylinder, steady-state

Where $N_0^{(1)}$ and $N_L^{(1)}$ are liner shape functions, by putting equation (2) in (1) and also $\delta_i = -1$ we have

$$\begin{aligned} N_0^{EDL,FVL} &= 2N_0^{(p)} - N_0^{(1)} \\ N_j^{EDL,FVL} &= 2N_j^{(p)} \\ N_L^{EDL,FVL} &= 2N_L^{(p)} - N_L^{(1)} \end{aligned} \qquad (3)$$

Equation (3) is non-oscillatory shape functions that will be used in this paper for CFD problems.

Examples

In this section, I am going to use the equation (3) as shape function for approximating a few equation and compare its result with non-constant δ_i. As first example, I approximated 1D advective equation by Point Collocation weight function, see figure (1). In second example, I approximated 1D convection-diffusion equation in $p_e = \infty$ by Galerkin weight function, see figure (2). In third example, I approximated 2D Euler equations for pressure distribution on surface of NACA 0012 airfoil [1] by Galerkin weight function and infinite elements for non-solid boundaries, see figure (4), and flow around a cylinder [1] by Subdomain Collocation weight function see figure (6).

Conclusions

As can be seen from the results, with any number of elements solutions are non-oscillatory, and when we use fine mesh (usually in CFD is used) solutions are very close together. So, using of constant EDL, FVL is useful.

References

1. Khaleghi M.R.A. A new computational package for using in CFD and other problems.