METHOD OF FORECASTING THE PROBABILITIES OF FUTURE CONSEQUENCES WHILE MAKING DECISIONS

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The article suggests a method of forecasting the probabilities of actualization of future events that are consequences of decisions, made by a subject. The method is based on combining the objective forecast of events that uses forecast data of former periods and subjective forecast that is received via subjective expert evaluations with sage of new relevant information. It has been shown that having statistic data on former expert forecasts on a relevant problem can help one define probabilities of future events, values of which do not depend on initial a priori probabilities and have objective character. Besides, a vector of objective probabilities is a eigenvector of an expert's forecast reliability matrix and corresponds to its unit eigenvalue. The method allows us to decrease a forecast's subjectivity greatly and increase its efficiency simultaneously.

Keyworks: making decisions, forecasting, probabilities, future events, eigenvector and eigenvalue

Within the process of making decisions we face a necessity to predict future results and consequences that can be caused by a variant of decision. A subject that makes a decision considers a set of possible consequences and results, and each of them can actualize in future, however, one doesn't know a priori which exact one and its probability. At the same time, adequacy of forecasting decisions' consequences by a subject defines achieving goals completely. Besides, it is necessary to consider limited abilities of a subject to adequately predict both future events and estimating of their probabilities [1].

In order to make the best and the most effective decisions it is necessary to possess scientific methods of forecasting. At the same time, there are no at present scientific methods of reliable and adequate prediction of future events and their probabilities. Most of the existing methods of forecasting are based on processing data of former predictions and on a baseless assumption that the past and the future are similar, therefore trends that were observed yesterday will preserve tomorrow and the day after. These methods can be used for short-term forecasts only with a condition that the environment, subject activity and their interaction do not suffer significant changes during the period. Of course, it can't be guaranteed. We cannot entrust to these methods when we require medium- and long-term forecast. In such cases possibilities of future events are absolutely indefinite and Bayesian correcting procedure of a priori probability to a posteriori probability is used to predict them [2, 3, 4]. Bayesian approach contains a set of arbitrary interpretations, and the received a posteriori probability still carries subjective nature.

In order to carry out scientific forecast at any time horizon it is necessary to possess methods that allow us to decrease subjectivity of forecasts and increase their objectivity level. These methods must combine both unavoidable subjective opinions, and objective information and statistic data that are known from former similar forecasts and relevant to the studied problem [5].

This article suggests a method that increases accurateness and reliability of prediction probabilities of future events, essentially. The method includes both subjective estimates and objective data of former periods if such exists. It has been shown that possession of data on reliability of the expert's former predictions can help us determinate objective probabilities of the predicted events that do not depend on a priori subjective probabilities estimates. We have received equations, solving which one can find objective probabilities of the forecasted events that are used to define the studied precise probabilities of the forecasted events.

A priori subjective probabilities of predicted events

Let us suppose that a subject or an expert are forecasting that after making a some decision, n consequences or events $A_1, A_2, \ldots, A_n$ can arise in future. The set (M) that consists of possible future events $A_1, A_2, \ldots, A_n$ must be as complete as possible so it is possible to suppose that one of these events will realize obligatory, that is $p(A_1) + p(A_2) + \ldots\ p(A_n) = 1$. Probabilities $p(A_1), p(A_2), \ldots, p(A_n)$ are not known to us a priori and are nominated by a subject according to his own comprehension of the moving events and, therefore, are subjective.

A subject, or an expert, realizing of arbitrariness of his subjective probabilities, instead of relying on them completely, decides to refer to another subject, or expert who is, in his opinion, a specialist in this area.

As the result of the taken research, the expert gives his set of probabilities of events $A_1, A_2, \ldots, A_n$. Opinions of the subject $A_{o,1}, A_{o,2}, \ldots, A_{o,n}$ on possibilities of any of future events $A_1, A_2, \ldots, A_n$ form a complete group. If the expert's opinion was absolutely true, than his claimed opinion $A_{o,i}$ that the event $A_i \in M$ will take place, it would guarantee its realization with the probability that equals 1. The expert's opinion $A_{o,i}$ can be both true, so that the event A_i will really takes place, and false so that another event $A_j \in M$ ($j = 1, 2, \ldots, n$) will take place instead of event *Ai*. It means that conditional probability of the expert's prediction of an event $A_{o,i}$ while really one of events $A_j \in M$ ($j = 1, 2, \ldots, n$) will take place, is measured with an value of conditional probability $p(A_{o,i}|A_j)$ that is a quantitative measure of expert's predictions accuracy and reflects its reliability.

Accuracy of expert's predictions that are expressed in conditional probabilities $p(A_{o,i}|A_j)$ can be received from information on similar predictions by this expert in past if such exist. Thus, for example, if from former experience we know that the expert's opinion $A_{o,i}$ on the realizing of event A_i was right for 75 % of cases, then error of his prediction equals $p(A_{o,i}|A_i) = 0{,}75$. And, if for 15 % of cases the expert predicted event A_i while event A_j, took place, then error of this expert's forecast equals $p(A_{o,i}|A_j) = 0{,}15$. Totality of all conditional probabilities $p(A_{o,i}|A_j)$, $i, j = 1, 2, \ldots, n$ describes the reliability level of forecast of the expert on this problem.

If such information is unavailable, or the studied event was not predicted before and are unique, then accurateness of predictions $p(A_{o,i}|A_j)$ by concrete expert are defined by subjective evaluations that reflect personal trust of the subject to the expert's opinion, or from control test evaluations of expert's qualification.

According to the formula of complete probability we receive a system of n equalities that define complete subjective probabilities $p(A_{o,i})$ of an expert that describe the degree of his certainty on actualization of an event A_i, that is

$$p(A_{o,i}) = \sum_{j=1}^{n} p(A_j)\, p(A_{o,i} \mid A_j), i = 1, 2, \ldots, n. \quad (1)$$

Probabilities $p(A_{o,i})$ specify a priori subjective probabilities $p(A_i)$ and are defined, on the one hand, by opinion of the expert or the subject, that is probabilities $p(A_j)$, $j = 1, 2, \ldots, n$, and, on the other hand, by the indicators of the expert's prediction accurateness that are defined independently of him and thus are objective.

Introducing vector columns

$$P(A_o) = \left(p(A_{o,1}), p(A_{o,2}), \ldots, p(A_{o,n})\right)^T;$$

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$$P(A) = \left(p(A_1), p(A_2), \ldots, p(A_n)\right)^T,$$

where T is transpose operation, and matrix S that consists of elements $p_{ij} = p(A_{\text{м},i}|A_j)$, $i, j = 1, 2, \ldots, n$,

$$S = \begin{pmatrix} p(A_{o,1} \mid A_1) & p(A_{o,1} \mid A_2) & \cdots & p(A_{o,1} \mid A_n) \\ p(A_{o,2} \mid A_1) & p(A_{o,2} \mid A_2) & \cdots & p(A_{o,2} \mid A_n) \\ \vdots & \vdots & \cdots & \vdots \\ p(A_{o,n} \mid A_1) & p(A_{o,n} \mid A_2) & \cdots & p(A_{o,n} \mid A_n) \end{pmatrix}, \quad (2)$$

we receive matrix equation of system (1):

$$P(A_o) = S \cdot P(A). \quad (3)$$

Elements $p_{ij} = p(A_{o,i}|A_j)$ of matrix S carry the information on accurateness of the probabilities of events predicted by the expert, and the complete matrix S describes the reliability of the expert's predictions regarding the studied events.

In matrix S each element, being a probability, is non-negative, and the sum of the elements of each column is equal to 1 due to the completeness of events $A_{o,1}, A_{o,2}, \ldots, A_{o,n}$. Elements of vectors $P(A)$ and $P(A_o)$ that are formed of probabilities of events that form a complete group are also non-negative and their sum is equal to 1. Matrix S that possesses the described characteristics, is called stochastic, and vectors $P(A)$ and $P(A_o)$ are called probability vectors [6].

Utmost objective probabilities of the predicted events

Probabilities $p(A_{o,i})$, $i = 1, 2, \ldots, n$, that are received from equality (1) or from its matrix analog (3) specify it's a priori subjective probabilities $p(A_i)$. Therefore, probabilities $p(A_{o,i})$ can again be taken as a priori probabilities and one can continue their specifying, implementing matrix equalities (3).

Let us assume that specified probabilities $P(A_o)$ that are calculated at the 1st stage of specification procedure, equal $P^{(1)}(A_o)$, and specification procedure is carried out by the same expert, so reliability of his forecasts regarding the studied events does not change and matrix S stays unaltered. Then, probability vector of events that has been specified at stage 2, $P^{(2)}(A_o) = S \cdot P^{(1)}(A_o)$. As according to (3) $P^{(1)}(A_o) = S \cdot P(A)$,, than $P^{(2)}(A_o) = S^2 \cdot P(A)$.

Procedure of specifying probabilities can be continued and at n-stage of specification we receive a probability vector $P^{(n)}(A_o)$ that is n-specification of initial a priori subjective probabilities $P(A)$:

$$P^{(n)}(A_o) = S^n \cdot P(A), \quad (4)$$

matrix S^n is n-degree of matrix S.

Process of specifying of probabilities that is carried out by the same expert, can be continued unlimitedly. We can prove [6] that for matrix equality (4) with stochastic matrix S and probability vector $P(A)$ the following statement is correct: vector $P^{(n)}(A_o)$ has a finite utmost vector $Q(A)$ while $n \to \infty$ that, first of all, is a probability vector, and, secondly, does not depends on values of a priori probabilities $P(A)$ and, thirdly, equals the right eigenvector of matrix S that corresponds to its maximum eigenvalue 1, in other words,

$$Q(A) = \lim_{n\to\infty} P^{(n)}(A_o) = \lim_{n\to\infty} S^n P(A) = S \cdot Q(A).$$

By this means column vector of utmost probabilities

$$Q(A) = \left(q(A_1), q(A_2), \dots, q(A_n)\right)^T$$

of the predicted events A_1, A_2, …, A_n equals proper eigenvector of stochastic matrix S that corresponds to eigenvalue 1 of this matrix, in other words,

$$Q(A) = S{\cdot}Q(A). \qquad (5)$$

Matrix system of equations (5) can be expressed in an expanded form:

$$q_i = \sum_{j=1}^{n} p_{ij} q_j,\ i = 1, 2, \dots, n, \qquad (6)$$

where $qi = q(Ai)$, $i = 1, 2, \dots, n$ are utmost probabilities of future events A_i.

In system (6) one of the equations of the system (actually, any of them) is linearly dependence on the other equations of the system, and, therefore, can be excluded. In order to the rest of the system of equations was compatible and have the unique solution, and the vector $Q(A)$ defined from the system of equations was probabilistic, it is necessary, instead of the excluded equation, join a normalization equality $q(A_1) + q(A_2) + \dots + q(A_n) = 1$ to system (6). It reflects the fact that the predicted events A_1, A_2, …, A_n form a complete group. By this means utmost probabilities $Q(A) = \left(q(A_1), q(A_2), \dots, q(A_n)\right)^T$ of actualization of future events A_1, A_2, …, A_n are defined from the system of linear equations (6) and added normalization equality.

Vector of probabilities $Q(A)$ represents an objective opinion of the expert regarding probabilities of the predicted events, to which he will come inevitably if he specifies his forecast regarding values of probabilities of events A_1, A_2, …, A_n repeatedly and, at most, unlimitedly. Besides, in order to specify, it is sufficient to possess the data on accurateness of an expert's predictions only. Objective nature of the utmost probabilistic vector $Q(A)$ comes from the received above fact, according to which, final probabilities $Q(A)$ do not depend on initial a priori subjective probabilities $P(A)$ that are set by a subject or an expert at the beginning of evaluation process. Therefore, utmost objective (UO) probabilities of the actualization of forecasted future events depend only on the degree of reliability of the expert's forecasts that is estimated by the matrix S of conditional probabilities $p(A_{o,i}|A_j)$. Matrix of forecasts' reliability S is defined not only by an expert's predictions' reliability, but also by the specific studied problem.

Specifying probabilities of the predicted events

Let us consider the method of prediction of probabilities of future events that uses both objective indicators of an expert's prediction accurateness that are expressed in UO probabilities, and subjective expert a priori estimations of probabilities of future events. And as we have established earlier, UO probabilities form a eigenvector that corresponds to the eigenvalue equals 1 of the stochastic matrix S of expert's forecasts.

Combining subjective and objective probabilities in the method is achieved via correction, or specifying of UO probabilities according to subjective opinion of the expert on the predicted events $A_{j,pr} \in M$. This correction is expressed by specified probabilities $q(A_i|A_{j,pr})$, $i, j = 1, 2, \dots, n$, of future event Ai on condition that the event $A_{j,pr} \in M$ will actualized.

After analyzing a new information, the expert predicts the following sequence of events: if future leads to realize the event $A_i \in M$ with UO probability $q(A_i)$, then the probability of new predicted event $A_{j,pr} \in M$, will equal conditional probability $p(A_{j,pr}|A_i)$, $i, j = 1, 2, \dots, n$. As the predicted event $A_{j,pr}$, $j = 1, 2, \dots, n$ actualizes in combination with one of events $A_i \in M$, $i = 1, 2, \dots, n$, then complete probability $p(A_{j,pr})$ can be expressed as:

$$p(A_{j,pr}) = \sum_{i=1}^{n} q(A_i)\, p(A_{j,pr} \mid A_i),$$

$$j = 1, 2, \dots, n.$$

Probability of a compatible actualizing two events $(A_{j,pr}, A_i)$, according to the formula of conditional probabilities, equals $p(A_{j,pr}, A_i) = p(A_{j,pr})\ q(A_i|A_{j,pr}) = q(A_i)\ p(A_{j,pr}|A_i)$. Thus, we receive the desired expression for a specified probability $q(Ai|Aj,np)$:

$$q(A_i \mid A_{j,pr}) = \frac{q(A_i)\, p(A_{j,pr} \mid A_i)}{p(A_{j,pr})},$$

$$i, j = 1, 2, \dots, n.$$

Specified probabilities $q(A_i|A_{j,pr})$ are the desired probabilities of future events $Ai \in M$ during the process of making decisions. They are defined both by objective UO probabilities $q(A_i)$ that are received according to the objective information on the expert's prediction of such events in the past, and by subjective probabilities $p(A_{j,pr}|A_i)$ of the forecasted events $A_{j,pr}$ on condition that only events A_i can actualize in future.

By this means the method of prediction values of probabilities of future events while making decisions combine both objective and subjective forecasting. Objective component of a forecast is based upon an objective data massive on an expert's predictions in the past on a relevant problem. This type of forecast is expressed with UO probabilities that, as it is shown in the article, are elements of eigenvector of stochastic matrix of predictions' reliability S that corresponds to its maximum eigenvalue, equals 1. Subjective forecast is carried out by subjective expert's estimations, according to analysis of new information on conditions of future. The method of prediction probabilities of future events that is described in the article, allows us to increase adequacy of forecasting future events significantly and decrease subjective component of forecasts, increasing their objective component at the same time.

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