Materials of Conferences

MODEL OF INTERACTION OF THE LAYER OF ICE AND NON-RIGID ROAD PAVEMENT WITH ADDITION OF THE RUBBER CRUMB

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Within last years the application of rubber granules as rubber -concrete filler is considered to be an advanced direction in road construction. Such kind of technology decreases the value of roadway covering and settles the issue of used car tyres utilization as well [4, 5]. Besides, field experience revealed a number of positive features of asphaltic surface: enhanced wear and cold resistance, increasing of service life, reduction of noise emission, shortening of breaking distance [6].

The purpose of this study is to determine mean stresses in the elements of ice, a loose road surface and tire under the action of the pressure of the wheels passing vehicles.

1. It is under consideration the area of wheel contact spot with rubber-concrete road covering, on the surface of which there is an ice layer. Spot size significantly exceeds rubber particles size, (NRRP) layers between them and thickness of ice layer. According to this fact evaluating the mode of deformation in the area of pressure spot for rubber-concrete-ice system, following assumptions will be reasonable:

a) There is no principle size for rubber particles, and its distribution in rubber-concrete covering is on average uniform ;

b) Ice thickness doesn't exceed average linear dimension of typical rubber-concrete volume;

c) Materials of structural elements (rubber, asphalt, ice) are isotropic an elastic;

d) Pressure of *p* wheel is constant on the contact spot;

e) Stress and deformation fields are uniform in all structural elements;

g) Structural elements become deformed together without breakaways before destruction.

In line with submitted assumptions it can be regarded that in the area of spot contact and wheel, the rubber-concrete – ice system appears as representative volume.

In accordance with assumption of physical fields uniformity, then average values are meant by stress and deformations. Two fragments can be emphasized in representative volume: rubber-ice (1) and NRRP -ice (2).

Keys:

σ_{11}, σ_{22}, σ_{33}, σ_{23}, σ_{13}, σ_{12} – average stress in representative volume;

ε_{11}, ε_{22}, ε_{33}, ε_{23}, ε_{13}, ε_{12} – average deformations in representative volume;

$\sigma^t_{11}, \sigma^t_{22}, \sigma^t_{33}, \sigma^t_{23}, \sigma^t_{13}, \sigma^t_{12}$ – average stress in *t* volume;

$\varepsilon^t_{11}, \varepsilon^t_{22}, \varepsilon^t_{33}, \varepsilon^t_{23}, \varepsilon^t_{13}, \varepsilon^t_{12}$ – average deformations in the same volume.

Thus super index indicates the volume, in which averaging has been carried out:

T = *p* – rubber particles;

t = *a* – NRRP elements;

t = *l*1 – ice elements contacting with rubber particle;

t = *l*2 – ice elements contacting with NRRP elements;

t = (1) – fragment (1) rubber-ice;

t = (2) – fragment (2) NRRP -ice.

There are stress and deformations matrixes

$$\{\sigma_{ij}\} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\}^T;$$

$$\{\varepsilon_{ij}\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{23}, \varepsilon_{13}, \varepsilon_{12}\}^T;$$

$$\{\sigma^t_{ij}\} = \{\sigma^t_{11}, \sigma^t_{22}, \sigma^t_{33}, \sigma^t_{23}, \sigma^t_{13}, \sigma^t_{12}\}^T;$$

$$\{\varepsilon^t_{ij}\} = \{\varepsilon^t_{11}, \varepsilon^t_{22}, \varepsilon^t_{33}, \varepsilon^t_{23}, \varepsilon^t_{13}, \varepsilon^t_{12}\}^T,$$

where index «T» means flip operation.

The object is issued: to define structural elements (micro stress) $\{\sigma^p_{ij}\}$, $\{\sigma^a_{ij}\}$, $\{\sigma^{л1}_{ij}\}$, $\{\sigma^{л2}_{ij}\}$ dependence on *p* wheel pressure.

Two averaging levels are assigned in representative volume: on the first level, averaging is carried out in each of fragments (1) and (2), regarded as two-component rubber-ice and NRRP -ice environments; on the second level, averaging is carried out in the whole representative element which regarded as two component environment containing (1) and (2) fragments.

Suggested idea of averaging method in [1] for two component environment is used while calling it into action in each level.

2. The averaging process for (1) and (2) fragments is under consideration. Conditions of equilibrium and ice and rubber displacement compatibility in (1) fragment can be recorded as follows:

$$\begin{cases} \sigma^{(1)}_{22} = \xi\sigma^{l1}_{22} + (1-\xi)\sigma^p_{22}, & \sigma^{(1)}_{11} = \sigma^{l1}_{11} = \sigma^p_{11}, \\ \sigma^{(1)}_{33} = \xi\sigma^{l1}_{33} + (1-\xi)\sigma^p_{33}, & \sigma^{(1)}_{13} = \sigma^{l1}_{13} = \sigma^p_{13}, \\ \sigma^{(1)}_{23} = \xi\sigma^{l1}_{23} + (1-\xi)\sigma^p_{23}, & \sigma^{(1)}_{12} = \sigma^{l1}_{12} = \sigma^p_{12}, \end{cases}$$

$$\begin{cases} \varepsilon^{(1)}_{11} = \xi\varepsilon^{l1}_{11} + (1-\xi)\varepsilon^p_{11}, & \varepsilon^{(1)}_{22} = \varepsilon^{l1}_{22} = \varepsilon^p_{22}, \\ \varepsilon^{(1)}_{13} = \xi\varepsilon^{l1}_{13} + (1-\xi)\varepsilon^p_{13}, & \varepsilon^{(1)}_{33} = \varepsilon^{l1}_{33} = \varepsilon^p_{33}, \\ \varepsilon^{(1)}_{12} = \xi\varepsilon^{l1}_{12} + (1-\xi)\varepsilon^p_{12}, & \varepsilon^{(1)}_{23} = \varepsilon^{l1}_{23} = \varepsilon^p_{23}. \end{cases} \quad (1)$$

$$\xi = \frac{\delta}{1+\delta}, \quad \delta = \frac{h}{a}. \qquad (2)$$

Parameter ξ – relative volume ice content in each of structural elements (1) and (2).

Mentioned equations reflect composition rule: component impact is proportional to its volume concentration; in this case equations placed in first columns of (1) and (2) systems, correspond to Reis's averaging, then to Foiht's averaging [2, 7].

In accordance with supposed assumptions state equations of ice and rubber materials are as follows:

$$\begin{cases} \varepsilon_{11}^{l1} = \frac{1}{E^l}\sigma_{11}^{l1} - \frac{\nu^l}{E^l}\sigma_{22}^{л1} - \frac{\nu^l}{E^l}\sigma_{33}^{l1}, & \varepsilon_{23}^{l1} = \frac{1}{G^l}\sigma_{23}^{l1}, \\ \varepsilon_{22}^{l1} = \frac{1}{E^l}\sigma_{22}^{l1} - \frac{\nu^l}{E^l}\sigma_{11}^{лl} - \frac{\nu^l}{E^l}\sigma_{33}^{l1}, & \varepsilon_{13}^{l1} = \frac{1}{G^l}\sigma_{13}^{l1}, \\ \varepsilon_{33}^{l1} = \frac{1}{E^l}\sigma_{33}^{l1} - \frac{\nu^l}{E^l}\sigma_{11}^{l1} - \frac{\nu^l}{E^л}\sigma_{22}^{l1}, & \varepsilon_{12}^{л1} = \frac{1}{G^l}\sigma_{12}^{l1}. \end{cases} \qquad (3)$$

$$\begin{cases} \varepsilon_{11}^{p} = \frac{1}{E^p}\sigma_{11}^{p} - \frac{\nu^p}{E^p}\sigma_{22}^{p} - \frac{\nu^p}{E^p}\sigma_{33}^{p}, & \varepsilon_{23}^{p} = \frac{1}{G^p}\sigma_{23}^{p}, \\ \varepsilon_{22}^{p} = \frac{1}{E^{lp}}\sigma_{22}^{p} - \frac{\nu^p}{E^{\,p}}\sigma_{11}^{p} - \frac{\nu^p}{E}\sigma_{33}^{p}, & {}_p\varepsilon_{13}^{p} = \frac{1}{G}\sigma_{13}^{p}, \\ \varepsilon_{33}^{p} = \frac{1}{E^p}\sigma_{33}^{p} - \frac{\nu^p}{E^p}\sigma_{11}^{p} - \frac{\nu^p}{E^p}\sigma_{22}^{p}, & \varepsilon_{12}^{p} = \frac{1}{G^p}\sigma_{12}^{p}. \end{cases} \qquad (4)$$

where E – Young's modulus; G – shear modulus; ν – Poisson number of ice (l) and rubber (p).

By the use of (1)–(4) equations, stresses in ice $\{\sigma_{ij}^{l1}\}$ and rubber $\{\sigma_{ij}^{p}\}$ elements can be expressed through $\{\sigma_{ij}^{(1)}\}$ stresses, functioning in (1) fragment in the whole:

$$\{\sigma_{ij}^{l1}\} = [P^{l1}]\{\sigma_{ij}^{(1)}\}; \ \{\sigma_{ij}^{p}\} = [P^{p}]\{\sigma_{ij}^{(1)}\}. \qquad (5)$$

Matrix $[P^{l1}]$ и $[P^p]$and have dimensions of 6x6, the elements involved are

$$p_{21}^{l1} = p_{31}^{l1} = \frac{B_1}{A_1 + A_2};$$

$$p_{21}^{p} = p_{31}^{p} = \frac{-\xi B_1}{(1-\xi)(A_1 + A_2)};$$

$$A_1 = \frac{1}{E^l} + \frac{\xi}{(1-\xi)E^p};$$

$$p_{22}^{л1} = p_{33}^{л1} = \frac{(A_1 + \nu^p A_2)B_2}{A_1^2 - A_2^2};$$

$$p_{22}^{p} = p_{33}^{p} = \frac{1}{1-\xi} - \frac{\xi(A_1 + \nu^p A_2)B_2}{(1-\xi)(A_1^2 - A_2^2)};$$

$$A_2 = -\frac{\nu^l}{E^l} - \frac{\xi\nu^p}{(1-\xi)E^p};$$

$$p_{23}^{л1} = p_{32}^{л1} = -\frac{(\nu^p A_1 + A_2)B_2}{A_1^2 - A_2^2};$$

$$p_{23}^{p} = p_{32}^{p} = \frac{\xi(\nu^p A_1 + A_2)B_2}{(1-\xi)(A_1^2 - A_2^2)};$$

$$B_1 = \frac{\nu^l}{E^l} - \frac{\nu^p}{E^p};$$

$$p_{44}^{l1} = \frac{G^l}{\varsigma G^l + (1-\varsigma)G^p};$$

$$p_{44}^{p} = \frac{G^p}{\varsigma G^l + (1-\varsigma)G^l};$$

$$B_2 = \frac{1}{(1-\xi)E^p}.$$

The remaining elements of the matrices $[P^{l1}]$ and $[P^p]$ and are equal to zero.

Further, having excluded from the equations (1)–(4) components of pressure and deformations in ice and rubber elements using the relation (5), we will receive the effective equation of a condition of a two-component fragment (1):

$$\{\varepsilon_{ij}^{(1)}\} = [S^{(1)}]\{\sigma_{ij}^{(1)}\}. \qquad (6)$$

Here the matrix $[S^{(1)}]$ size is 6×6, its elements are:

$$s_{11}^{(1)} = \frac{\xi}{E^л} + \frac{1-\xi}{E^p} + 2\left(\frac{\xi\nu^p}{(1-\xi)E^p} - \frac{\nu^л}{E^л}\right)\frac{B_1}{A_1 + A_2};$$

$$s_{12}^{(1)} = s_{13}^{(1)} = -\frac{\nu^p}{E^p} - \frac{\xi(1-\nu^p)B_1 B_2}{A_1 + A_2};$$

$$s_{22}^{(1)} = s_{33}^{(1)} = B_2 - \frac{\xi B_2^2}{A_1^2 - A_2^2}((1+(\nu^p)^2)A_1 + 2\nu^p A_2);$$

$$s_{21}^{(1)} = s_{31}^{(1)} = -\frac{\nu^p}{E^p} - \frac{\xi(1-\nu^p)B_1^2}{A_1 + A_2};$$

$$s_{23}^{(1)} = s_{32}^{(1)} = -\nu^p B_2 + \frac{\xi B_2^2}{A_1^2 - A_2^2}(2\nu^p A_1 + \left(1+(\nu^p)^2\right)A_2);$$

$$s_{44}^{(1)} = \frac{1}{\xi G^л + (1-\xi)G^p}; \quad s_{55}^{(1)} = s_{66}^{(1)} = \frac{\xi}{G^л} + \frac{1-\xi}{G^p}.$$

he remaining elements of the matrix $[S^{(1)}]$ are zero.

For pressure and deformations in NRRP and ice elements in a fragment (2) the same equations, as well as (1)–(4) for rubber and ice in a fragment (1) are fair. Therefore they can be received, if in (1)–(4) to replace indexes of sizes under the scheme:

$$(1) \rightarrow (2);\ l1 \rightarrow l2;\ p \rightarrow a.$$

As consequence, from (5) we receive dependences of pressure in asphalt elements $\left\{\sigma_{ij}^{a}\right\}$ and ice $\left\{\sigma_{ij}^{l2}\right\}$ through pressure $\left\{\sigma_{ij}^{(2)}\right\}$, operating on a fragment (2) in whole, in a kind,

$$\left\{\sigma_{ij}^{l2}\right\} = [P^{l2}]\left\{\sigma_{ij}^{(2)}\right\};$$

$$\left\{\sigma_{ij}^{a}\right\} = [P^{a}]\left\{\sigma_{ij}^{(2)}\right\}, \qquad (7)$$

and from (6) – the effective equation of a condition of a two-component fragment (2):

$$\left\{\varepsilon_{ij}^{(2)}\right\} = [S^{(2)}]\left\{\sigma_{ij}^{(2)}\right\}. \qquad (8)$$

3. For representative volume as the two-component environment consisting of fragments (1) and (2), the equations of balance and compatibility of deformations are fair

$$\begin{cases} \sigma_{11} = \rho\sigma_{11}^{(1)} + (1-\rho)\sigma_{11}^{(2)}, & \sigma_{33} = \sigma_{33}^{(1)} = \sigma_{33}^{(2)}, \\ \sigma_{22} = \rho\sigma_{22}^{(1)} + (1-\rho)\sigma_{22}^{(2)}, & \sigma_{23} = \sigma_{23}^{(1)} = \sigma_{23}^{(2)}, \\ \sigma_{12} = \rho\sigma_{12}^{(1)} + (1-\rho)\sigma_{12}^{(2)}, & \sigma_{13} = \sigma_{13}^{(1)} = \sigma_{13}^{(2)}, \end{cases} \qquad (9)$$

$$\begin{cases} \varepsilon_{33} = \rho\varepsilon_{33}^{(1)} + (1-\rho)\varepsilon_{33}^{(2)}, & \varepsilon_{11} = \varepsilon_{11}^{(1)} = \varepsilon_{11}^{(1)}, \\ \varepsilon_{23} = \rho\varepsilon_{23}^{(1)} + (1-\rho)\varepsilon_{23}^{(2)} & \varepsilon_{22} = \varepsilon_{22}^{(1)} = \varepsilon_{22}^{(1)}, \\ \varepsilon_{13} = \rho\varepsilon_{13}^{(1)} + (1-\rho)\varepsilon_{13}^{(2)} & \varepsilon_{12} = \varepsilon_{12}^{(1)} = \varepsilon_{12}^{(1)}. \end{cases} \qquad (10)$$

where ρ is the relative volume concentration of rubber in rubber-concrete, defined as

$$\rho = \frac{a}{a+b}.$$

From the equations (6), (8)-(10) we will express pressure $\left\{\sigma_{ij}^{(1)}\right\}$, $\left\{\sigma_{ij}^{(2)}\right\}$ in fragments (1) and (2) through pressure $\{\sigma_{ij}\}$, operating on representative volume in whole, in a kind

$$\left\{\sigma_{ij}^{(1)}\right\} = [P^{(1)}]\left\{\sigma_{ij}\right\};$$

$$\left\{\sigma_{ij}^{(2)}\right\} = [P^{(2)}]\left\{\sigma_{ij}\right\}. \qquad (11)$$

Matrices have the size of 6×6, the elements involved are:

$$p_{1j}^{(1)} = \frac{1}{\Delta}(a_{22}b_{1j} - a_{12}b_{2j});$$

$$p_{2j}^{(1)} = \frac{1}{\Delta}(a_{11}b_{2j} - a_{21}b_{1j});$$

$$p_{ij}^{(2)} = \frac{1}{1-\rho}\left(\delta_{ij} - p_{ij}^{(1)}\right);$$

$$i = 1, 2, j = 1, 2, 3,$$

$$p_{66}^{(1)} = \frac{s_{66}^{(2)}}{\rho s_{66}^{(2)} + (1+\rho)s_{66}^{(1)}};$$

$$p_{66}^{(2)} = \frac{s_{66}^{(1)}}{\rho s_{66}^{(2)} + (1+\rho)s_{66}^{(1)}};$$

$$p_{ii}^{(1)} = p_{ii}^{(2)} = 1;$$

$$i = 3, 4, 5,$$

$$a_{i1} = s_{i1}^{(1)} + \frac{\rho}{1-\rho}s_{i1}^{(2)}; \quad a_{i2} = s_{i2}^{(1)} + \frac{\rho}{1-\rho}s_{i2}^{(2)};$$

$$b_{ij} = \frac{1}{1-\rho}(\delta_{1j} + \delta_{2j})s_{ij}^{(2)} + \delta_{3j}\left(s_{i3}^{(2)} - s_{i3}^{(1)}\right).$$

The remaining elements of the matrix $[P^{(1)}]$ и $[P^{(2)}]$and and are equal to zero; δ_{ij}– Kroneker's symbol.

Further, by means of (11) it is excluded from the equations (6), (8)-(10) components of pressure and deformations in fragments (1) and (2), we will receive the effective equation of a condition of a material of *representative volume:*

$$\{\varepsilon_{ij}\}=[S]\,\{\sigma_{ij}\}. \qquad (12)$$

Here the matrix [*S*] is of dimension 6×6; its elements are given by:

$$s_{1j} = s_{11}^{(2)}p_{1j}^{(2)} + s_{12}^{(2)}p_{2j}^{(2)} + \delta_{3j}p_{13}; \quad s_{2j} = s_{21}^{(2)}p_{1j}^{(2)} + s_{22}^{(2)}p_{2j}^{(2)} + \delta_{3j}p_{23};$$

$$j = 1, 2, 3\; j = 1, 2, 3,$$

$$s_{3j} = \rho\left(s_{31}^{(1)}p_{1j}^{(1)} + s_{32}^{(1)}p_{2j}^{(1)} + \delta_{3j}p_{33}^{(1)}\right)+(1-\rho)\left(\left(s_{31}^{(2)}p_{1j}^{(2)} + s_{32}^{(2)}p_{2j}^{(2)} + \delta_{3j}p_{33}^{(2)}\right);$$

$$s_{44} = \rho s_{44}^{(1)} + (1-\rho)s_{44}^{(2)}; \quad s_{55} = \rho s_{55}^{(1)} + (1-\rho)s_{55}^{(2)};$$

$$s_{66} = \frac{s_{66}^{(1)}s_{66}^{(2)}}{\rho s_{66}^{(1)} + (1-\rho)\, s_{66}^{(2)}}.$$

The remaining elements of the matrix [*S*] are zero.

4. Let's consider, that in a zone of a stain of contact in representative volume conditions are satisfied:

1) $\sigma_{11} = -p$; $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$.

Where *p* – pressure of a wheel of the car;

2) Deformations in a direction of axes of coordinates 2 and 3 are completely constrained:

$$\varepsilon_{22} = 0,\ \varepsilon_{33} = 0.$$

Under these conditions from the equation (15) it is found

$$\sigma_{22} = -p_{21}p; \quad \sigma_{33} = -p_{31}p.$$

Where

$$p_{21} = \frac{s_{31}s_{23} - s_{21}s_{33}}{s_{22}s_{33} - s_{23}s_{32}};$$

$$p_{31} = \frac{s_{21}s_{32} - s_{22}s_{31}}{s_{22}s_{33} - s_{23}s_{32}}.$$

Hence, the matrix of average pressure in representative volume looks like:

$$\{\sigma_{ij}\}=\{\sigma\}\cdot p. \qquad (13)$$

Where

$$\{\sigma\}=\{-1 \quad p_{21} \quad p_{31} \quad 0 \quad 0 \quad 0\}^T..$$

As a result, according to the equations (5), (7), (11), (13) pressure in elements of ice, rubber and asphalt are connected with pressure of a wheel in a zone of a stain of contact by the equations

$$\{\sigma_{ij}^{л1}\}=[P^{л1}]\,[P^{(1)}]\,\{\sigma\}p;$$

$$\{\sigma_{ij}^{л2}\}=[P^{л2}][P^{(2)}]\{\sigma\}p;$$

$$\{\sigma_{ij}^{p}\}=[P^{p}][P^{(1)}]\{\sigma\}p;$$

$$\{\sigma_{ij}^{a}\}=[P^{a}]\,[P^{(2)}]\,\{\sigma\}p.$$

Thus, the task in view is solved.

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HIGH-TORQUE ELECTRIC ENGINE FOR THE MOTOR-WHEEL

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In the modern world the economy of industrial resources has got the greatest value. To save on energy is possible only by construction of the electrical machines, that best of all satisfy the requirements for them by these systems.

The constantly growing use of a municipal motor-vehicle transport has led to the necessity of development of the machines which are not pollute air pools by exhaust gases, have low noise level and progressive constructive decisions. Modern technical systems have a number of lacks in the technical and economic parameters due to forces of friction [1]. The new concept of motor-wheel drive ex-