Materials of Conferences

ON STUDIES OF NON-LINEAR OSCILLATIONS IN THE SYSTEM OF SECOND ORDER DIFFERENTIAL EQUATIONS WITH SLOWLY FLUCTUATING COEFFICIENTS

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In this work conditions of solutions' existence that are described by a significantly non-linear system of the second order of the following type with slowly altering coefficients:

$$\begin{aligned}\dot{x} &= X_o^{(m)}(x,y) + \mu f_1(x,y,\tau,\mu);\\ \dot{y} &= Y_o^{(m)}(x,y) + \mu f_2(x,y,\tau,\mu),\end{aligned} \qquad (1)$$

where

$$X_o^{(m)}(x,y) = A_o x^m + A_1 x^{m-1} y + $$
$$+ \ldots + A_{m-1} x y^{m-1} + A_m y^m,$$
$$Y_o^{(m)}(x,y) = B_o x^m + B_1 x^{m-1} y + \ldots + B_{m-1} x y^{m-1} + B_m y^m,$$
$$A_i,\ B_i\ (i = 1, 2, \ldots, m)$$

– are constant coefficients, and

$$\begin{aligned} f_1(x,y,\tau,\mu) &= X_1^{(m_1)}(x,y,\tau) + \mu X_2^{(m_2)}(x,y,\tau) + \ldots + \mu^{k-1} X_k^{(m_k)}(x,y,\tau);\\ f_2(x,y,\tau,\mu) &= Y_1^{(m_1)}(x,y,\tau) + \mu Y_2^{(m_2)}(x,y,\tau) + \ldots + \mu^{k-1} Y_k^{(m_k)}(x,y,\tau),\end{aligned} \qquad (2)$$

$X_k^{(m_k)}(x,y,\tau)$, $Y_k^{(m_k)}(x,y,\tau)$ – are multinomials reative to x, y of any final degree of m_k and they do not contain terms of lower than m – order with coefficients that are limited functions on c τ and with the first derivatives, limited on τ, μ is unlimitedly small parameter, $\tau = \mu t$ is slow time.

Let us imply that right parts of the system (1) equal zero only in the point $x = y = 0$.

The problem study goes with the function of Lyapunov, via method, introduced by G.V. Kamenkov [1].

First of all, let us show a topological study of so-called «shortened» system. Shortened system that corresponds to initial equations (1) has the following type:

$$\begin{aligned}\dot{x} &= \frac{dx}{dt} = X_o^{(m)}(x,y);\\ \dot{y} &= \frac{dy}{dt} = Y_o^{(m)}(x,y).\end{aligned} \qquad (3)$$

Questions of qualitative theory of stability of differential equations according to Lyapunov were studied by G.V. Kamenkov in 1935.

G.V. Kamenkov showed [1] that behavior of integral curves around the coordinates beginning of two functions:

$$\begin{aligned} x\,Y_o^{(m)}(x,y) - y\,X_o^{(m)}(x,y) &= F(x,y);\\ x\,X_o^{(m)}(x,y) + y\,Y_o^{(m)}(x,y) &= R(x,y)\end{aligned} \qquad (4)$$

depending on structure of these functions can be described in a number of ways. Let us say that equation $F(x, y) = 0$ has substantial roots (each substantial root of this equation defines a curve that, together with axis OX, forms an angle, tangent of which equals k) $k = \frac{y}{x}$, and form $R(x, y)$, while $F(x, y) = 0$ preserves constant sense for any k. This event is characterized by a knot. Besides, if $R(x, y) > 0$, this knot is unstable, and if $R(x, y) < 0$ it is stable. If equation $F(x, y) = 0$ has substantial roots, and form $R(x, y)$ on one of the beams equals more than zero, and on other beams is less, then such event is characterized as a «saddle».

Let us imply now that $F(x, y)$ is defined in sense.

Let us write down (4) in polar coordinates:

$$\begin{aligned}\dot{r} &= \frac{dr}{dt} = r^m R_o^{(m+1)}(\theta);\\ \dot{\theta} &= \frac{d\theta}{dt} = r^{m-1} F_o^{(m+1)}(\theta),\end{aligned} \qquad (5)$$

where

$$\begin{aligned} R_o^{(m+1)}(\theta) &= \bar{X}_o^{(m)}(\cos\theta, \sin\theta)\cos\theta + \bar{Y}_o^{(m)}(\cos\theta, \sin\theta)\sin\theta;\\ F_o^{(m+1)}(\theta) &= -\bar{X}_o^{(m)}(\cos\theta, \sin\theta)\sin\theta + \bar{Y}_o^{(m)}(\cos\theta, \sin\theta)\cos\theta.\end{aligned} \qquad (6)$$

The solution of the system (5) looks as:

$$r = r_o \exp \int_0^{\theta} \frac{R_o^{(m+1)}(\theta)}{F_o^{(m+1)}(\theta)} d\theta. \qquad (7)$$

Thus, in formula (7) under-integral expression is a periodical function of θ, then (7) it can be described as:

$$r = r_o \exp\left(\frac{a_0}{2} + \sum_{n=1}^{\infty} \frac{a_n}{n}\sin n\theta - \frac{b_n}{n}\cos n\theta\right), \qquad (8)$$

where a_o, b_o, a_n, b_n are coefficients of degradation of functions $R_o^{m+1}(\theta) / F_o^{m+1}(\theta)$ into the line of Fourier.

It is clear that (8) is an equation of spirals. If $a_o > 0$, they are unstable. If $a_o = 0$, phase trajectories form locked curves, and the beginning of coordinates is the center. Periodic solution of the system (3) corresponds to the letter event.

Thus, conditions of existence of periodic solutions for system (3) comes to the fulfillment of two requirements:

a) Function $F(x, y)$ must be defined in sense;

b) $a_o = \frac{1}{\pi}\int_0^{2\pi}\frac{R_o^{(m+1)}(\theta)}{F_o^{(m+1)}(\theta)}d\theta = 0.$

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ABOUT A BOUNDARY-VALUE PROBLEM OF STURM-LIOUVILLE WITH NOT SEPARABLE BOUNDARY CONDITIONS OF THE FIRST TYPE

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Let's consider differential operator Sturm-Liouville of the second order:

$$-y''(x)+q(x)\cdot y(x)=\lambda\cdot a^2\cdot y(x),\quad 0\le x\le\pi,\quad a>0, \tag{1}$$

with not separable boundary conditions of the first type (see [1]):

$$\begin{cases} y'(0)+a_{11}\cdot y(0)+a_{12}\cdot y(\pi)=0,\\ y'(\pi)+a_{21}\cdot y(0)+a_{22}\cdot y(\pi)=0, \end{cases} \tag{2}$$

where $a_{km}\in C\quad (k,m=1;\,2)$, and it is supposed that potential $q(x)$ – summable function on the segment $[0;\pi]$:

$$q(x)\in L_1[0;\pi](=)\left(\int_0^x q(t)dt\right)'_x = q(x) \tag{3}$$

almost everywhere on $[0;\pi]$.

Theorem. Asymptotics of the eigenvalues of the differential operator (1)–(2) with a condition (3) has the following kind:

$$s_k=\frac{k}{a}+\frac{d_{1k}}{ak}+\frac{d_{2k}}{ak^2}+O\left(\frac{1}{k^3}\right),\quad k=1,2,3,... \tag{4}$$

and for this

$$d_{1k}=\frac{1}{2\pi}\cdot\left[\int_0^\pi q(t)dt+\int_0^\pi q(t)\cos(2kt)dt-2(a_{11}-a_{22}+a_{12}-a_{21})\right], \tag{5}$$

$$d_{2k}=-\frac{d_{1k}}{2\pi}\cdot\int_0^\pi(2t-\pi)q(t)\cdot\sin(2kt)dt+\frac{a_{11}-a_{22}}{2\pi}\cdot\int_0^\pi q(t)\sin(2kt)dt- \\ -\frac{1}{4\pi}\cdot\int_0^\pi q(t)\cdot\left(\int_0^t q(\zeta)\cdot\left[\sin(2k\zeta)-\sin(2kt)-\sin(2k(\zeta-t))\right]\cdot d\zeta\right)dt,... \tag{6}$$

The theorem is proved by methods of the chapter 5 of the monograph [2].

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THE MATERIAL WORLDS HIERARCHY EMPIRICAL MODELS

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As, now, it was known, on the basis of all those natural science models, having described in the author's papers [1, 2] etc, having taking into account the physicists' empirical conclusions, the findings, and the experimental results after Albert Einstein, the STEREOCHRONODYNAMICS objective reasons had been noted – the physical theory, that could be created the time – space mathematical model, which should to be had the quite necessary and the sufficient flexibility in the time – space all the properties description, including the modern physical