Materials of conferences

SOLUTION OF THE MAIN MIXED PROBLEM OF THE ELASTICITY THEORY FOR A HALF-PLANE WITH CURVED BOUNDARY

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Let us analyze semi-infinite Region S in Plane Z.

The Function, mapping the lower half-plane ImZ<0 onto the investigated Region S, is accepted in the form of a polynomial, as it was done in the work [1]:

$$z = \omega(\zeta) = A + B\zeta - \sum_{k=0}^{n} \frac{c_{2k+1}}{(\zeta + a - bi)^{2k+1}} \tag{1}$$

where $z = x + iy, \zeta = \xi + i\eta, \eta < 0, A, B, c_1, c_3, \ldots c_{2n+1}, a, b > 0$ are real numbers.

Let us assign the boundary values of Displacement Component $g_1(t)$ and $g_2(t)$ at Segment L' of Boundary L of Region S, occupied by an elastic body, and at the rest Segment L'' – boundary values of Stress Component $N(t)$ and $T(t)$. It is necessary to determine the stressed state of Region S.

Along with the given problem we will analyze the problem of stressed state determination, presuming that the main external force vectors $(X_k, Y_k), k = 1, \ldots n$ are assigned, which are applied to every Segment L_k separately.

We will refer to the above-formulated problems as problems A and B, according to [2]. Thus to obtain a common solution of the given problems let us use the methods developed by N.I. Muskhelishvili [2], as well as in the paper of I.N. Kartsivadze [3]. Let us extend here the approach of the author of paper [3] to the case of a half-plane, subjected to conformal mapping.

As it is known [2], the formulas, expressing the displacement and stress components through Functions $\Phi(\zeta)$ and $\Psi(\zeta)$ of Complex Variable ζ take the form of

$$Y_\eta + X_\xi = 2\left\{\Phi(\zeta) + \overline{\Phi(\zeta)}\right\} = 4\,\mathrm{Re}\,\Phi(\zeta), \tag{2}$$

$$Y_\eta - X_\xi + 2iX_\eta = \frac{2}{\overline{\omega'(\zeta)}}\left\{\overline{\omega(\zeta)}\,\Phi'(\zeta) + \omega'(\zeta)\Psi(\zeta)\right\}, \tag{3}$$

Summing up the left and right parts of these equations, and, then proceeding to conjugate values, we get

$$Y_\eta - iX_\eta = \Phi(\zeta) + \overline{\Phi(\zeta)} + \frac{1}{\omega'(\zeta)}\left\{\omega(\zeta)\overline{\Phi'(\zeta)} + \overline{\omega'(\zeta)}\;\overline{\Psi(\zeta)}\right\} \tag{4}$$

The formula, expressing Displacement Components u and v in Cartesian coordinates, is given by

$$2\mu(u + iv) = \aleph\varphi(\zeta) - \omega(\zeta)\overline{\Phi(\zeta)} - \overline{\psi(\zeta)}, \tag{5}$$

where: $\varphi'(\zeta) = \Phi(\zeta)\omega'(\zeta), \quad \psi'(\zeta) = \Psi(\zeta)\omega'(\zeta).$

If Functions $\varphi(\zeta)$ and $\psi(\zeta)$ are assigned, then Functions $\Phi(\zeta)$ and $\Psi(\zeta)$ are entirely determined. And in case Functions $\Phi(\zeta)$ and $\Psi(\zeta)$ are assigned, Functions $\varphi(\zeta)$ and $\psi(\zeta)$ are determined with accuracy to arbitrary constants. Therefore Eq.(5) may be written in the form of

$$2\mu(u + iv) = \aleph\varphi(\zeta) - \omega(\zeta)\overline{\Phi(\zeta)} - \overline{\psi(\zeta)} + const. \tag{6}$$

Assuming now, that $\omega(\zeta)$ is a rational function, let us apply Function $\Phi(\zeta)$ determination to Region $\mathrm{Im}\zeta > 0$, supposing that

$$\omega'(\zeta)\Phi(\zeta) = -\omega'(\zeta)\overline{\Phi}(\zeta) - \omega(\zeta)\overline{\Phi}'(\zeta) - \overline{\omega}'(\zeta)\overline{\Psi}(\zeta), \tag{7}$$

where: ξ lies in the upper half-plane.

This operation must be carried out so that the values of Function $\Phi(\zeta)$, determined in the upper half-plane, can be analytically extended to the lower half-plane through the unoccupied segments of boundaries (if there are any).

Taking into account (7), having changed ζ for $\bar{\zeta}$ and proceeded to conjugate values, we obtain

$$\omega'(\zeta)\Psi(\zeta) = -\overline{\omega'}(\zeta)\bar{\Phi}(\zeta) - \bar{\omega}(\zeta)\Phi'(\zeta) - \overline{\omega'}(\zeta)\Phi(\zeta) = \\ = -\overline{\omega'}(\zeta)\{\Phi(\zeta) + \bar{\Phi}(\zeta)\} - \bar{\omega}(\zeta)\Phi'(\zeta). \tag{8}$$

Eq. (8) expresses the values of $\Psi(\zeta)$ for the lower half-plane through values of $\Phi(\zeta)$, for the lower half-plane, as well as for the upper half-plane.

The determination of Function $\varphi(\zeta)$ can be extended to the upper half-plane, provided that the relation holds in the upper half-plane

$$\varphi(\zeta) = \int \Phi(\zeta)\omega'(\zeta)d\zeta.$$

Integrating both parts of expression (7) over ζ, and, dropping the arbitrary constant value, we have

$$\varphi(\zeta) = -\bar{\Phi}(\zeta)\omega(\zeta) - \bar{\psi}(\zeta) \tag{9}$$

where: ξ lies in the upper half-plane.

Similarly,

$$\psi(\zeta) = -\Phi(\zeta)\bar{\omega}(\zeta) - \bar{\varphi}(\zeta) \tag{10}$$

where: ξ lies in the lower half-plane.

Thus, Stress and Displacement Components can be expressed through one and the same Function $\Phi(\zeta)$, determined in the lower half-plane, as well as for the upper half-plane.

To fulfill further solution it is necessary to employ Expression (2) and Expression (4), taken in the form of:

$$Y_\eta - iX_\eta = \Phi(\zeta) - \Phi(\bar{\zeta}) + \left\{\frac{\omega(\zeta)}{\omega'(\zeta)} - \frac{\omega(\bar{\zeta})}{\omega'(\bar{\zeta})}\right\}\overline{\Phi'(\zeta)} + \left\{\frac{\overline{\omega'(\zeta)}}{\omega'(\zeta)} - \frac{\overline{\omega'(\zeta)}}{\omega'(\bar{\zeta})}\right\}\overline{\Psi(\zeta)}, \tag{11}$$

It is necessary to explain that Expression (11) is obtained after Function $\Phi(\bar{\zeta})$ is subtracted from the right part of (4) and after the same function replaced with its expression obtained from Eq. (8) after having proceeded to conjugate values is added there. Let us notice that the expression determined by Eq. (8) is meant by Function $\Psi(\zeta)$.

For Displacement Components u and v, replacing $\psi(\zeta)$ in Eq. (6) with expression (10), we obtain the following relation

$$2\mu(u + iv) = \aleph\varphi(\zeta) + \varphi(\bar{\zeta}) - \{\omega(\zeta) - \omega(\bar{\zeta})\}\overline{\Phi(\zeta)} + const.$$

From this point on we will also need the expression for $u' + iv'$, where,

$$u' = \frac{\partial u}{\partial \xi}, \quad v' = \frac{\partial v}{\partial \xi}.$$

It can be obtained if we differentiate both parts of Eq. (5) over ξ. Then we have

$$2\mu(u' + iv') = \aleph\omega'(\zeta)\Phi(\zeta) - \omega'(\zeta)\overline{\Phi(\zeta)} - \omega(\zeta)\overline{\Phi'(\zeta)} - \overline{\omega'(\zeta)\Psi(\zeta)}.$$

Having added Function $\omega'(\zeta)\Phi(\bar{\zeta})$ to the right part of above relation and subtracted the same Function, replaced with its expression, obtained from Eq. (8) after proceeding to conjugate values, we have

$$2\mu(u' + iv') = \omega'(\zeta)\{\aleph\Phi(\zeta) + \Phi(\bar{\zeta})\} - \omega'(\zeta)\left\{\frac{\omega(\zeta)}{\omega'(\zeta)} - \frac{\omega(\bar{\zeta})}{\omega'(\bar{\zeta})}\right\}\overline{\Phi'(\zeta)} -$$

$$-\omega'(\zeta)\overline{\omega'(\zeta)}\left\{\frac{1}{\omega'(\zeta)}-\frac{1}{\omega'(\bar{\zeta})}\right\}\overline{\Psi(\zeta)}. \tag{12}$$

Let us note that both of Functions $\Phi(\zeta)$ and $\Psi(\zeta)$ are holomorphic in the lower half-plane.

Now it is necessary to ascertain the behaviour of Function $\Phi(\zeta)$, extended by Eq. (7) to the upper half-plane. To do that it is enough to analyze the behaviour of Product $\Phi(\zeta)\omega'(\zeta)=\varphi'(\zeta)$, referring to Eq. (7) which determines Function $\Phi(\zeta)$ when $\operatorname{Im}\zeta>0$.

Now let us proceed to the analysis of problems A and B.

Let us denote the points of Real Axis $O\xi$ which correspond to the points a_k, b_k of the Cutout L as α_k, β_k, a segment of a real line which corresponds to L' as $O\xi'$, and the rest segment of the line as $O\xi''$.

As we know [1] the solution of the first main problem, it is more convenient to take account of the influence of the forces assigned at Segment L'' separately. Consequently, it can be supposed that Segment L' of Boundary L is free from external stresses.

Then, based on Eqs. (11) and (12) the boundary conditions in both problems are given by

$$\Phi^+(t)-\Phi^-(t)=0 \quad на \quad L'', \tag{13}$$

$$[\omega'(t)\Phi(t)]^+ + \aleph[\omega'(t)\Phi(t)]^- = 2\mu g'(t) \quad на \quad L' \tag{14}$$

Eq. (13) proves that Segment $O\xi''$ of the real axis is not saltus function for Function $\Phi(\zeta)$, i.e. that Function $\Phi(\zeta)$ is a holomorphic one on a cut-down plane $O\xi'$ except for finite number of points where it may have poles. The same refers to Function $\omega'(\zeta)\Phi(\zeta)$.

To determine it, let us refer to Eq. (14) representing a boundary condition of the problem well-known in the theory of functions of a complex variable, which is Riemann problem or Gilbert problem. It is completely investigated in the works of N.I. Muskhelishvili [4], F.D. Gakhov [5] and other authors. N.I. Muskhelishvili calls it "boundary value problem of linear conjugation", or "conjugate problem" for short.

Assuming that Function $\omega'(\zeta)\Phi(\zeta)$ may have poles of order not higher than $m_1, m_2, \ldots m_l, m$ in Points $\zeta_1, \zeta_2, \ldots \zeta_l, \infty$ and employing the results of the conjugate problem solution [2, p.397], we obtain for the sought function

$$\omega'(\zeta)\Phi(\zeta)=\frac{\mu X_0(\zeta)}{\pi i}\int_{L'}\frac{g'(t)}{X_0(t)(t-\zeta)}dt + X_0(\zeta)R(\zeta), \tag{15}$$

where:

$$X_0(\zeta)=\prod_{j=1}^{r}(\zeta-a_j)^{-\gamma}(\zeta-b_j)^{\gamma-1},\ \gamma=\frac{1}{2}-\beta i, \beta=\frac{\ln\aleph}{2\pi}. \tag{16}$$

with $X_0(\zeta)$ implying such a branch that $\lim\limits_{\zeta\to\infty}\zeta^n X_0(\zeta)=1$, and $R(\zeta)$ is a rational function of the form

$$R(\zeta)=\sum_{j=1}^{l}\left\{\frac{C_{j1}}{(\zeta-\zeta_j)}+\frac{C_{j2}}{(\zeta-\zeta_j)^2}+\cdots+\frac{C_{jm_j}}{(\zeta-\zeta_j)^{m_j}}\right\}+P(\zeta), \tag{17}$$

where: $P(\zeta)$ is a polynomial of degree not higher than $m+r$.

Coefficients included in Expression (17) are determined based on the additional conditions of Problems A and B.

Let us consider an example of a stamping tool with a footing parallel to axis $O\xi$, provided that the stamping tool moves only vertically, so

$$g'(t) = 0 \quad on \quad L' \tag{18}$$

Besides, suppose that external forces influencing the stamping tool have a resultant force directed vertically downwards, so

$$X = 0, \quad Y = -p, \tag{19}$$

where: p is a positive constant set in advance.

In this case Eqs. (7) and (8) take the form of

$$\omega'(\zeta)\Phi(\zeta) = -\left(B + \sum_{k=0}^{n} \frac{(2k+1)c_{2k+1}}{(\zeta + a - bi)^{2k+2}}\right)\overline{\Phi}(\zeta) - \left(A + B\zeta - \sum_{k=0}^{n} \frac{c_{2k+1}}{(\zeta + a - bi)^{2k+1}}\right)\overline{\Phi}'(\zeta) -$$
$$-\left(B + \sum_{k=1}^{n} \frac{(2k+1)c_{2k+1}}{(\zeta + a + bi)^{2k+2}}\right)\overline{\Psi}(\zeta) \qquad npu \quad \zeta \in \operatorname{Im}\zeta > 0, \tag{20}$$

$$\omega'(\zeta)\Psi(\zeta) = -\left(B + \sum_{k=0}^{n} \frac{(2k+1)c_{2k+1}}{(\zeta + a + bi)^{2k+2}}\right)\overline{\Phi}(\zeta) - \left(A + B\zeta - \sum_{k=0}^{n} \frac{c_{2k+1}}{(\zeta + a + bi)^{2k+1}}\right)\Phi'(\zeta) -$$
$$-\left(B + \sum_{k=0}^{n} \frac{(2k+1)c_{2k+1}}{(\zeta + a + bi)^{2k+2}}\right)\Phi(\zeta) \qquad npu \quad \zeta \in \operatorname{Im}\zeta < 0. \tag{21}$$

as for rational Function (1) there are poles of order $1,3,5,\ldots,2n+1$ in Point $\zeta = -a + bi$ and at infinity, and for Function $\omega'(\zeta)$ there are poles of order $2,4,6,\ldots,2n+2$ in the same point $\zeta = -a + bi$. Thus, for Function $\omega'(\zeta)\Phi(\zeta)$ there are poles of order not higher than $2,4,6,\ldots,2n+2$ in Point $\zeta = -a + bi$. Assuming that $\Phi(\zeta)$ together with $\omega'(\zeta)\Phi(\zeta)$ disappear at infinity, according to (15) and taking into account (18), we obtain

$$\omega'(\zeta)\Phi(\zeta) = \left(C_0 + \frac{D_1}{(\zeta + a - bi)^2} + \frac{D_2}{(\zeta + a - bi)^4} + \cdots + \frac{D_n}{(\zeta + a - bi)^{2n+2}}\right)X_0(\zeta) \tag{22}$$

where: $D_1, D_2, \ldots, D_n$ are to be determined.

According to (16)

$$X_0(\zeta) = (\zeta + l)^{-\frac{1}{2}+i\beta}(\zeta - l)^{-\frac{1}{2}-i\beta}$$

For large values of $|\zeta|$ it takes the form of

$$X_0(\zeta) = \frac{1}{\zeta} + \frac{\alpha}{\zeta} + \ldots, \tag{23}$$

where: $\alpha = 2\beta l i.$

Note that Function $\Psi(\zeta)$ corresponding to Function $\Phi(\zeta)$ that is determined both in the lower and in the upper half planes, should be holomorphic in the lower half plane. But this is not the case, because according to (21) it has a pole in Point $\zeta = -a + bi$.

Constants $D_1, D_2, \ldots, D_n$ are determined based on the holomorphy of Function $\Psi(\zeta)$ in Point $\zeta = -a + bi$. Denote them as $D_1^*, D_2^*, \ldots, D_n^*$.

Now it is necessary to figure out constant value C_0. Let us remark here that for large values of $|\zeta|$ it follows from (22) and (23) that

$$\omega'(\zeta)\Phi(\zeta) = \frac{C_0}{\zeta} + o\left(\frac{1}{\zeta}\right).$$

On the other side, Function $\Phi(\zeta)$ for large values of $|\zeta|$ [2, p.339] involves

$$\omega'(\zeta)\Phi(\zeta) = -\frac{X+iY}{2\pi}\cdot\frac{1}{\zeta} + o\left(\frac{1}{\zeta}\right),$$

therefore

$$C_0 = -\frac{X+iY}{2\pi} \qquad (24)$$

Thus, Function $\omega'(\zeta)\Phi(\zeta)$ that is set by Eq. (22) is completely determined.

Applying (19) and taking into account (24) result in

$$\omega'(\zeta)\Phi(\zeta) = \left(\frac{i\,p}{2\pi} + \frac{D_1^*}{(\zeta+a-bi)^2} + \frac{D_2^*}{(\zeta+a-bi)^4} + \cdots + \frac{D_n^*}{(\zeta+a-bi)^{2n+2}}\right)X_0(\zeta). \qquad (25)$$

Formulas for estimating pressure $P(t)$ and tangential stress $T(t)$, influencing the body under the stamping tool, are analogous to those of classical case and take the form of

$$\omega'(t)\{P(t)+iT(t)\} = \frac{\aleph+1}{\aleph}\left[\omega'(t)\,\Phi(t)\right] \qquad (26)$$

From this, taking into account Eq. (22), it follows that

$$\omega'(t)\{P(t)+iT(t)\} = \frac{\aleph+1}{\aleph}\left(\frac{ip}{2\pi} + \frac{D_1^*}{(t+a-bi)^2} + \frac{D_2^*}{(t+a-bi)^4} + \ldots\right.$$

$$\left.\ldots + \frac{D_n^*}{(t+a-bi)^{2n+2}}\right)(t+l)^{-\frac{1}{2}+i\beta}(t-l)^{-\frac{1}{2}-i\beta}. \qquad (27)$$

Now, separating the real and the complex parts it is possible to obtain formulas for pressure $P(t)$ and tangential stress $T(t)$.

In conclusion let us emphasize the following circumstance: if we assume for Function (1) that $B=1$, and put to zero all the other coefficients, we obtain a function that allows self-reflecting of the half-plane.

In this case Function (25) becomes

$$P(t)+iT(t) = \frac{i\,p}{2\pi}\frac{\aleph+1}{\aleph}(t+l)^{-\frac{1}{2}+i\beta}(t-l)^{-\frac{1}{2}-i\beta}. \qquad (28)$$

From this it is easy to get formulas well-known in mathematical theory of elasticity, which allow solving a classic problem of a stamping tool with a linear horizontal footing

$$P(t) = \frac{p}{\pi\sqrt{l^2-t^2}}\frac{1+\aleph}{\sqrt{\aleph}}\cos\left[\frac{\ln\aleph}{2\pi}\ln\frac{l+t}{l-t}\right], \qquad (29)$$

$$T(t) = \frac{p}{\pi\sqrt{l^2-t^2}}\frac{1+\aleph}{\sqrt{\aleph}}\sin\left[\frac{\ln\aleph}{2\pi}\ln\frac{l+t}{l-t}\right],$$

that was obtained independently by V.M. Abramov [6] who used the method of integral transformations and by N.I. Muskhelishvili [2] who applied methods of the theory of functions of a complex variable.

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NOISE AT SAW AND WOODWORKING INDUSTRIES IN RUSSIA: FROM THEORY AND EXPERIMENTS TO PRODUCTION SECTOR WANTS

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The noise deteriorating effect on the efficiency of labour and human health is commonly known. In the past decades the noise quieting problem in developed nations of the world, and our country as well, has become a top-ranking one.

The noise effect on the worker's body goes beyond the influence on the organ of hearing only. Hygienists have found out that in some noisy branches the general morbidity rate rises by (10…15)%. It is proved that even the levels of (40…70) dBA have an effect on the vegetative nervous system irrespective of the subjective noise perception of the person. The habituation of the person to noise is illusive, as noise affects even the sleeping person.

The action of noise is often coupled with the effect of other destructive for human health factors: vibrations, irradiations, dust and gas content, etc. It also accentuates the requirements to restrict noise exposure and promotes the untimely retirement benefits.

It must be emphasized that the USSR was the first in the world to initiate the noise attack by law (The Resolution of the Council of Ministers of the USSR 1960, 1969, 1973).

In 1971 the "Sanitary standards of industrial institutions" CH 245 – 71 and "Hygienic standards of sound pressure and volume tolerance levels at work places" ГH 1004 – 73 were developed. The Committee of Measures and Active Measuring Instruments at the Council of Ministers of the USSR established a series of Noise National Standards included into the "System of Labour Safety Standards". Nowadays the sanitary norms CH 2.2.4/2.1.8.562 – 96 work, where the noise norm for working places makes 80 dBA on the sound level.

The requirement toughening for noise from 90 dBA (according to CH 245 – 71) up to 80 dBA forced us to work in 3 directions in this problem solution [1 – 6].

1. The investigation of noise, causes and noise making objective laws of active processing equipment as part of processing lines and departments. The saw and woodworking equipment is, as is known, characterized by a high efficiency, at which one has to appoint process speeds from 40 to 100 (and even faster) m/sec with feed velocities up to 150 m/min.

Let us note that the quantity of working tools (saws, shaft arbors) to provide the efficiency achieves the number of 4-10 and more in one machine. For the moment of our works starting (meeting of 60-70s of the past century) there was no slightest hint in the operating equipment and technological designs to any noise reducing solutions in the constructions, working areas, plant and seliteb (by-plant) territories. On the active processing equipment, at the minimal interference with the construction, we developed the devices with due consideration of rather various methods of noise control: the acoustic suppression in the generation places, sound protection and absorption, acoustic shielding and noise localization methods. A noticeable (5…10 dBA) effect was obtained. In some instances we managed to interfere with the mainstream technology as well, when the equipment location was determined by the technology requirements only. The shop drawings of the devices were sent by us in a great amount at the request of numerous enterprises (not only saw and woodworking industries).

The dominant requirement, which would exclude the rejection of antinoise devices by the workers, was sustained by us in the direction of technological capabilities non-reduction of a machine or aggregate.

2. The results of theoretic and experimental research have served the foundation for the leading engineering materials complex development for production workers, design engineers, original equipment (methods, guidance, instructions, drawing albums) manufacturers. All of them have come through the stages of reconciliation with the State design institutes of the branch, production engineering design consultancies, main engineering department on woodworking processing equipment design, Research and development institutes of the branch and original equipment manufacturers, trade union Central Committee of the branch before the approval of the Ministry of Forest Industry. So that the design engineers "were not afraid" of acoustic designs (and it would be a novelty for them), we carried out a week training. The following years of communication with engineering designers testified that necessary theoretical knowledge on noise, methods of design policy for all possible technological situations, types of shop floors and equipment stock, illustrated with plenty of numerical examples, allow them to lead acoustic designs without any difficulties.

With the appearance of data-flow computers we created the computation algorithms and programs