1) For any finite limited value l in the point a of this p.a. function F(z) any generating

this point sequence (z_n) is made up of a part of the collection of sequences

$$\left(z^{(j)}\left(x_n^{(j)},t_n^{(j)}\right)\right)$$
 with $a=\infty$, or

$$\left(\frac{1}{z^{(j)}\left(x_n^{(j)}, t_n^{(j)}\right)} + a\right) \text{ with } a \in \mathbf{C}, \quad (5)$$

being obtained by substitution into the expressions (4) of some convergent real sequences $(x_n^{(j)})$ and $(t_n^{(j)})$; $(t_n^{(j)})$ converging to some finite, and $(x_n^{(j)})$ - to infinite limits;

2) Vice versa: for any convergent real-valued sequences (x_n) and (t_n) , where (t_n) converges to an arbitrary finite, and (x_n) - to infinite limits, every of the sequences (5) generates a finite limited value of the p.a. function F(z) in its isolated singular 1-point $a \in \overline{\mathbb{C}}$ (with $a = \infty$ or $a \in \mathbb{C}$ accordingly).

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CLUSTER SETS OF BIANALYTIC FUNCTIONS IN THEIR ISOLATED SINGULARITIES

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Brief

General methods of finding an explicit cluster set of any bi-analytic function in its arbitrary isolated singularity.

1. Let it be required to find a cluster set C(F(z),a) [1-6] of an arbitrary bi-analytic function (b.a. function) [2-6]

$$F(z) = f_1(z)\overline{z} + f_2(z) \tag{1}$$

in its isolated singular point $a \in \overline{\mathbb{C}}$, where the functions $f_1(z)$ and $f_2(z)$ are arbitrary analytic functions in a deleted neighborhood O(a) of the point $a \in \overline{\mathbb{C}}$ (so called analytic components of the b.a. function F(z)).

The first step in finding C(F(z),a) should be most reasonable to make determining the fact if the point $a \in \overline{\mathbb{C}}$ is an isolated singularity at least for one of the functions – analytic components of the function F(z), as the following result [2-4] makes sense.

Lemma 1

If the point $a \in \overline{\mathbb{C}}$ is an essential isolated singularity for at least one of the functions $f_j(z)$ (j=1,2), then C(F(z),a) is total (i.e. coincides with $\overline{\mathbb{C}}$, where F(z) in a b.a. function (1) given in a deleted neighborhood O(a) of the point a.

2. Lemma 1 allows considering only those cases, when the isolated singularity $a \in \overline{\mathbf{C}}$ of the b.a. function F(z) is not an essential singularity for $f_1(z)$ and $f_2(z)$; besides, in this case we are always successful in converting finding C(F(z),a) to finding a cluster set in the point ∞ of a bi-analytic first-degree polynomial (i.e. a function from the ring $\mathbf{C}[z,\overline{z}]$) relative \overline{z} or a function of the

form $\frac{\overline{z}}{z}$ [5, 6, 10]. The following rather

simple properties will help such changing.

Property 1

where $a,b \in \mathbb{C}$ and $a \neq 0$.

For any function w(z) given in an unlimited set $D(w) \subset \mathbb{C}$ (i.e. $\infty \in \overline{D(w)}$) and for any polynomial $p(z) \in \mathbb{C}[z] \setminus \mathbb{C}$ $C(w(z), \infty) = C(w(p(z)), \infty)$, in particular $C(w(z), \infty) = C(w(az + b), \infty)$,

Property 2

For any function w(z) given in an unlimited set $D(w) \subset \mathbb{C}$ and for any polynomial

$$p(z) = a_n z^n + ... + a_1 z + a_0 \in \mathbb{C}[z],$$

 $a_n \neq 0, n \in \mathbb{N}$, the congruence is valid:

$$C(p(z) \cdot w(z), \infty) = C(a_n z^n \cdot w(z), \infty).$$

Property 3

For any defined and continuous in \mathbb{C} function w(z), any point $a \in \overline{\mathbb{C}}$ and any function j(z) of complex variable z the congruence is valid:

$$C(w(j(z)),a)\setminus\{\infty\}=w(C(j(z),a))\setminus\{\infty\},$$

in particular, if w(z) is an arbitrary polynomial P(z) from $\mathbb{C}[z] \setminus \mathbb{C}$, then

$$C(P(j(z)),a) = P(C(j(z),a)),$$

where j(z) is a function defined in a deleted

neighborhood O(a) of the point $a \in \overline{\mathbb{C}}$ (a natural agreement that $P(\infty) = \infty$ being accepted).

Property 4

For any function F(z) with $D(F) \subset \overline{\mathbb{C}}$ and any $a \in \mathbb{C}$ the congruence is valid: $\mathbb{C}(F(z), a) = C(F(z+a), 0)$.

Points

Property 4 evidently always allows passing on from the research of a cluster set of an arbitrary b.a. function in its arbitrary isolated singularity $a \in \overline{\mathbf{C}}$ to the research of a cluster set of a b.a. function in its isolated singularity $a = \infty$ or a = 0; besides, a transition to any of these two variants is always possible [10]. Let us further suppose that $a = \infty$.

Property 5

For any complex-valued functions $w_1(z)$ and $w_2(z)$ with unlimited

$$\begin{split} D(w_1) &= D(w_2) \subset \mathbf{C} \,, \text{ if } \\ C(w_2(z), \infty) &= \{a\} \quad \text{with} \quad a \in \mathbf{C} \,, \quad \text{then } \\ C(w_1(z) + w_2(z), \infty) &= C(w_1(z), \infty) + a \,; \quad \text{if } \\ a \neq 0 \,, \text{ then } \end{split}$$

$$C(w_1(z) \cdot w_2(z), \infty) = a \cdot C(w_1(z), \infty).$$

The following fact arises from Property 5 (generalizing of Property 2).

Property 6

For any complex-valued functions $w_1(z)$, $w_2(z)$ and $w_3(z)$ defined in one and the same unlimited set of the plane \mathbb{C} , if

$$\lim_{z\to\infty}\frac{w_1(z)}{w_2(z)}=1,$$

then $C(w_1(z) \cdot w_3(z), \infty) = C(w_2(z) \cdot w_3(z), \infty)$.

Property 7

For any complex-valued functions $w_1(z)$, $w_2(z)$ and $w_2(z)$ defined in an unlimited set of the plane \mathbb{C} , if

$$\lim_{z\to\infty}\frac{w_1(z)}{w_2(z)}=\infty,$$

then

$$C((w_1(z) + w_2(z)) \cdot w_3(z), \infty) = C(w_1(z) \cdot w_3(z), \infty)$$

The analogous property can be formulated for complex-valued functions of two real variables as well.

Property 8

For any complex-valued functions A(x, y), B(x, y) and F(x, y) of two real variables x

and y with coinciding domains, for which the point $(a_1,a_2)\in \overline{R}^2$ is a cluster one, if

$$\lim_{x \to a_1, y \to a_2} \frac{B(x, y)}{A(x, y)} = 0,$$
then

$$C((A(x, y) + B(x, y)) \cdot F(x, y), (a_1, a_2) = C(A(x, y) \cdot F(x, y), (a_1, a_2))$$

3. Let F(z) be a b.a. function of the form (1) given in a deleted neighborhood of the point $a = \infty$; ∞ point not being an essential singularity for analytic components of this b.a.

function, then the following representation in some $\overset{o}{O}(\infty)$ is valid for it:

$$F(z) = F_{\infty}(z) + F_{W}(z) + F_{0}(z), \tag{2}$$

where

$$F_{\infty}(z) = (a_k z^k + \dots + a_1 z + a_0) \overline{z} + (b_m z^m + \dots + b_1 z);$$

$$F_{w}(z) = a_{-1} z^{-1} \overline{z} + b_0;$$

$$F_{0}(z) = (a_{-2} z^{-2} + a_{-3} z^{-3} + \dots) \overline{z} + (b_{-1} z^{-1} + b_{-2} z^{-2} + \dots),$$
(3)

where $a_k, ..., a_0, a_{-1}, ...$ and

 $b_m,\ldots,b_0,b_{-1},\ldots$ are corresponding Laurent series expansion coefficients in the neighborhood of the point $a=\infty$ of the analytic components $f_1(z)$ and $f_2(z)$ of the b.a. function F(z), i.e.

$$f_1(z) = \sum_{j=k}^{-\infty} a_j z^j$$
, where

$$k \in \mathbb{Z}, \ a_j \in \mathbb{C}(j = k,...,0,-1,...);$$

$$f_2(z) = \sum_{j=m}^{-\infty} b_j z^j$$
 , where

$$m \in \mathbb{Z}, \ b_j \in \mathbb{C}(j = m, ..., 0, -1, ...)$$

As it is evident that $\lim_{z\to\infty}F_0(z)=0$, then $C(F(z),\infty)=C(F_\infty(z)+F_W(z),\infty)$ and it is natural to consider two possible situations for $F_\infty(z)$:

a) F_{∞} (z) $\equiv 0$ and then $C(F(z),\infty) = C(F_w(z),\infty) = a_{-1}w + b_0$, where $W = \{z \in \mathbb{C} \mid \mid z \mid = 1\}$ is a unit circle of the plane \mathbb{C} , $a_{-1} \neq 0$. In this case ∞ point for F(z) is called an isolated singular O-point [6-8]. But if $F_{\infty} \equiv 0$ and, moreover, $a_{-1} = 0$, then $C(F(z),\infty) = \{b_0\}$ and ∞ point is called a removable isolated point of the function F(z) [6-8].

b) $F_{\infty}(z) \not\equiv 0$. In this case the properties of this b.a. polynomial $F_{\infty}(z)$ will be determining in the structure $C(F(z),\infty)$; the influence of the function F_{w} on $C(F(z),\infty)$ will be equal to a parallel translation of the set $C(F_{\infty}(z),\infty)$ [10].

Lemma 2

Let a bi-analytic in a deleted neighborhood $O(\infty)$ function F(z) be represented in this domain in the form (2) (besides for the corresponding function F_∞ : $a_k \neq 0$, $k \in N_0$, then for availability of finite elements in the cluster set $C(F(z), \infty)$ it is necessary that:

- 1) k+1=m;
- 2) $|a_k| = |b_m|$;
- 3) but if $k \in N$, and $a_k = b_m = 1$, then $b_{m-1} \in R$.

Points

It is obvious that for any b.a. function given in some $\stackrel{o}{O}(\infty)$, if the corresponding to it function $F_\infty\not\equiv 0$, then $\infty\in C(F(z),\infty)$ always.

Complementation

Let for a b.a. function F(z) the corresponding to it function $F_{\infty}(z) \not\equiv 0$ satisfies the conditions

(1)-(2) of Lemma 2, then, if $C(F(z), \infty)$ contains finite elements, then any sequence (z_n) generating any of them satisfies the following condition:

$$\lim_{n\to\infty}\frac{\overline{z}_n}{z_n}=-\frac{b_m}{a_n}.$$

Points

The facts which are analogous to Lemma2 and the complementation to it, are obviously valid for the isolated singularity a=0 as well; besides it is easy to see that the substitution of $F_w(z)$ by the corresponding constant and the substitution of

$$z$$
 in $F_{\infty}(z) \not\equiv 0$ by the function $\frac{1}{z}$ with

multiplying function $F_{\infty}\!\!\left(\frac{1}{z}\right)$ by the function of

the form $A\frac{\overline{z}}{z}$ (where $A\in \mathbf{C}$ is selected properly [10]) allows passing on from C(F(z),a) to $C(\widetilde{F}(z),\infty)$, where $\widetilde{F}(z)$ is a b.a. function defined in a deleted neighborhood $O(\infty)$, and moreover

 $\widetilde{F}(z)$ is a b.a. polynomial from $\mathbb{C}[z,\overline{z}]$.

4. The following two lemmas will give us an opportunity to substantiate the theorem of Sokhotsky-Weierstrass type for bi-analytic functions fully. Besides it is worth underlying that the above points allow further formulating of many facts without loss of reasoning generality only for the case of isolated singular ∞ point of b.a. polynomials.

Lemma 3

For any bi-analytic polynomial $p(z, \overline{z})$ from $\mathbb{C}[z, \overline{z}]$ there is a certain augmented with ∞ point straight line of the plane $\overline{\mathbb{C}}$, in which there are all points from $C(p(z, \overline{z}), \infty)$.

Lemma 4

Let it for a bi-analytic polynomial

$$p(z,\bar{z}) = (a_k z^k + ... + a_1 z + a_0) \bar{z} + b_m z^m + ... + b_1 z + b_0 \in \mathbb{C}[z,\bar{z}] \setminus \mathbb{C},$$

where $k, m \in N_0$, $a_k \neq 0$, be known that $l \in \mathbb{C}$ is any its limited in ∞ point value, which is generated by a sequence (z_n) , then every sequence of the form

$$(\widetilde{z}_n(t)) = \left(z_n + \frac{t}{z_n^k}\right) t \in \mathbb{C},$$

generates a finite point

$$l + b_m \cdot \left(t - \left(-\frac{a_k}{b_m} \right)^m \cdot \bar{t} \right),$$

of the cluster set $C(p(z, \overline{z}), \infty)$.

Complementation

If $p(z, \overline{z}) \in \mathbb{C}[z, \overline{z}] \setminus \mathbb{C}$, then $C(p(z, \overline{z}), \infty)$ either a straight line (augmented by ∞ point) or a singleton $\{\infty\}$.

The following theorem of Sokhotsky-Weierstrass type was formulated in [4], but without its full substantiating [5, 6, 10].

Theorem 1

A cluster set in any point $a \in \overline{\mathbb{C}}$ of any b.a. function (1)

 $F(z)=f_1(z)\overline{z}+f_2(z)$, given in a deleted neighborhood $\overset{o}{O}(a)$ of this point a either consists of one point of the plane $\overline{\mathbf{C}}$, or represents a circle of the plane $\overline{\mathbf{C}}$, or is total, i.e. coincides with $\overline{\mathbf{C}}$. Vice versa, for any point $a\in\overline{\mathbf{C}}$ and any set $\Phi\subset\overline{\mathbf{C}}$ which is

either a circle, or a straight line (augmented with ∞ point), or a one-element set (i.e. $\Phi = \{c\}$ with $c \in \overline{\mathbb{C}}$), or is the whole plane $\overline{\mathbb{C}}$, there is a defined in a deleted neighborhood O(a) of the point a bi-analytic function F(z), for which $C(F(z),a) = \Phi$.

5. Let us obtain the criterion which allows distinguishing if the point $a \in \mathbb{C}$ is a pole for the b.a. function given in a O(a) (i.e. $C(F(z),a)=\{\infty\}$ or C(F(z),a) is a straight line of the plane $\overline{\mathbb{C}}$.

To do it let us pass on to the search of a cluster set in ∞ point corresponding to the function F(z) and, in general terms, an arbitrary bianalytic polynomial $p(z,\bar{z}) \in \mathbf{C}[z,\bar{z}] \setminus \mathbf{C}$ of the form (3), as it is precisely this situation which a general case with $F_{\infty}(z) \not\equiv 0$ is converted to; considering, of course, that the conditions k+1=m and $|a_k|=|b_m|$ are hold.

1) Having applied Property 1 with $b=0,\ a=a$ to $p(z,\overline{z})$, we'll infer:

$$C(p(z,\bar{z}),\infty) = C(p(az,a\bar{z}),\infty);$$

supposing further that
$$a = \sqrt{\frac{a_k}{b_{k+1}}}$$

(we'll mean any of its two values by the radical), and then applying Property 3 we infer:

$$C\left(p\left(z,\overline{z}\right),\infty\right)=a^{k+1}b_{k+1}C\left(p_{1}(z,\overline{z}),\infty\right) \text{ ,where } \\ p_{1}(z,\overline{z})=(z^{k}+a_{k-1}^{(1)}z^{k-1}+...+a_{0}^{(1)})\overline{z}+(z^{k+1}+b_{k}^{(1)}z^{k}+...+b_{1}^{(1)}z+b_{0}^{(1)}) \text{ .} \\ \text{Applying Properties 2 and 5 to } p_{1}(z,\overline{z}) \text{ we infer: } \\$$

$$C(p_1(z,\bar{z}),\infty) = C(z^k\bar{z} + z^{k+1} + b_k^{(2)}z^k + ... + b_1^{(2)}z + b_0^{(2)},\infty),$$

where $z^{k+1} + b_k^{(2)} z^k + ... + b_1^{(2)} z + b_0^{(2)}$ is an integral part of a rational function

$$\frac{z^{k+1} + b_k^{(1)} z^k + \dots + b_1^{(1)} z + b_0^{(1)}}{z^k + a_{k-1}^{(1)} z^{k-1} + \dots + a_0^{(1)}} z^k.$$

If k = 0, then we get immediately that

$$C(a_0\bar{z} + b_1z + b_0, \infty) = \sqrt{\frac{a_0}{b_1}}b_1C(z + \bar{z}, \infty) + b_0 = \{\sqrt{a_0b_1} \cdot t + b_0 \mid t \in \overline{R}\},$$

and all the sequences (z_n) generating all finite points from $C(p(z, \overline{z}), \infty)$ are of the following form:

$$(z_n) = \left((x_n + iy_n) \sqrt{\frac{a_0}{b_1}} \right),$$

where (x_n) and (y_n) are arbitrary real-valued sequences possessing the property:

$$x_n \to \frac{t}{2} \in R$$
, and $y_n \to \infty$.

2) Denoting $p_2(z, \overline{z}) = z^k \overline{z} + z^{k+1} + b_k^{(2)} z^k + ... + b_1^{(2)} z + b_0^{(2)}$ $(k \ge 1)$ and passing on to real variables x and y, we infer

$$C(p_2(z,\bar{z}),\infty) = C\left(p_2(x+iy,x-iy),\left(-\frac{b_k^{(2)}}{2},\infty\right)\right)$$

because (see Lemma 2) the coefficient $b_k^{(2)}$ can be thought as a real number (if not $C(p(z,\bar{z}),\infty)=\{\infty\}$).

3) Having substituted the variables $x_1 = x + \frac{b_k^{(2)}}{2}$, we'll infer

$$C(p_2(z,\overline{z}),\infty) = C \left(p_2 \left(x_1 - \frac{b_k^{(2)}}{2} + iy, x_1 - \frac{b_k^{(2)}}{2} - iy \right) (0,\infty) \right)$$

and having applied not more than k-1 times Property 8 [5, 10], we'll "delete" all the monomials containing x_1 symbol (in the corresponding true degree), excluding x_1y^k , and we'll get that

$$C(p_2(z,\bar{z}),\infty) = 2i^k C(x_1 y^k + p(y),(0,\infty)).$$

4) Dividing the real and imaginary parts of the polynomial

$$p(y) = P_1(y) + iP_2(y)$$
, where

 $P_1(y), P_2(y) \in R[y]$

(besides, on its proper inception $\deg P_1(y) < k$), we infer the following theorem [see 10]:

Theorem 2

Let $p(z, \overline{z})$ be a nontrivial polynomial of the form (3). Then $C(p(z, \overline{z}), \infty)$ has finite elements when and only when $p_2 = P_2(y) \equiv const$, in addition

$$C(p(z,\overline{z}),\infty) = \left\{ 2i^k \left(\sqrt{\frac{a_k}{b_{k+1}}} \right)^{k+1} \cdot b_{k+1} \cdot (t+ip_2) \middle| t \in \overline{R} \right\};$$

besides, all the sequences generating all finite points from $C(p(z, \overline{z}), \infty)$, are given by the following formula:

$$(z_n) = (x_n + iy_n) = \left(\left(\frac{t_n - P_1(y_n)}{y_n^k} - \frac{b_k^{(2)}}{2} + iy_n \right) \sqrt{\frac{a_k}{b_{k+1}}} \right)$$

where (t_n) , (y_n) are arbitrary real-valued sequences, and also (t_n) converges to the finite limit t, and (y_n) - to ∞ .

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PROJECTING COURSE IN MATHEMATICS ON THE IDEA OF INTERSUBJECT COMMUNICATIONS' GENERALIZATION

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In new social and economical conditions Higher School teachers face the necessity of specialists' training improving on the basis of integral combination of professional education with the high level of their fundamental training.

Mathematical Education in a Higher Technical School has the development of cognitive abilities at such a level of acquirement which could provide conscious using methods of routine problems' solution and research, their translation onto a non-routine problem (the problem with technical-grade content), i.e. on the level of conscious translation of mental activity approaches onto training objects of engineering disciplines, onto techniques and technology which will be used in the future work activity, as one of its basic aims.

The analysis of mathematical content of general professional disciplines shows that the definition of interactions between all courses of the curriculum of a Higher School represents a complex enough task. First of all, let us answer the question: Is it worth performing interactions between all the disciplines of the curriculum in equal amount? The absence of a clear answer to the question on practice leads to the fact that there is often a tendency to establish connections between all the disciplines of the curriculum of a Higher School in equal amount.

A specially carried out research and study of advanced pedagogical experience showed that