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ABOUT SOME METHODS OF CONSTRUCTION POLYANALYTIC FUNCTIONS WITH A PREDETERMINED LIMIT SET IN THEIR ISOLATED SINGULAR L-POINT

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In this paper we consider a new method of quick construction of polyanalytic functions with a predetermined cluster set in isolated singular B-points (radiant points) of these functions.

It is known [1–5, 7] that if a set $M \subset \overline{\mathbf{C}}$ of any of the four types is given: an extended complex plane $\overline{\mathbf{C}}$, a polynomial image $P(w)$ of a unit circle $w = \{z \in \mathbf{C} : |z| = 1\}$ ($P(z) \in \mathbf{C}[z] \setminus \mathbf{C}$), an arbitrary unit subset of the set $\overline{\mathbf{C}}$, and finally, a union of finite number of nondegenerate polynomial lines $\bigcup_{j=1}^{m} L_j$ augmented with ∞ point, then for any point $a \in \overline{\mathbf{C}}$ there is such a deleted neighborhood $\overset{0}{O}(a)$ of its and a defined in it poly-analytic (p.a.) function $F(z)$, i.e. the function of the kind

$$F(z) = \sum_{k=0}^{n-1} f_k(z)\bar{z}^k, \qquad (1)$$

where $n \in N$, $f_k(z)$ ($k = 0,..., n-1$) – are analytic in $\overset{0}{O}(a)$ functions, that $C(F(z), a) = M$. Let us remind that the number $n \in N$ is called [1] the order of poly-analytic property of the function $F(z)$, and if

$f_{n-1} \not\equiv 0$, then it is called the proximate order of its poly-analyticity; the functions $f_k(z)$ ($k = 0,..., n-1$) are called the analytic components of the poly-analytic (or as it is spoken about, n- analytic) function $F(z)$.

However, the earlier offered in [2,3] method of finding the corresponding p.a. function for the last, the fourth, case, when the point a is called [2, 3] the isolated singular l-point of the function $F(z)$, and the predetermined set $M \subset \overline{\mathbf{C}}$ is sure to have the form

$$M = \bigcup_{j=1}^{m} L_j = \bigcup_{j=1}^{m} P_j(\overline{R}), \qquad (2)$$

where $L_j = P_j(\overline{R})$, $P_j(z) \in \mathbf{C}[z] \setminus \mathbf{C}$, $P_j(\infty) = \infty$; $j = 1,..., m$, unlike the rest of the cases, was very complicated and tedious.

In this article essentially more simple modes of construction of a p.a. function possessing a limit set of the kind (2) in its isolated l-point are offered, $P_j(z) \in \mathbf{C}[z] \setminus \mathbf{C}$ ($j = 1,..., m$) being arbitrary predetermined polynomials different from those identical to the constants.

As compelling for any $a \in \mathbf{C}$ congruence $C(F(z), a) = C(F(z+a), 0)$ allows considering that $a = 0$ or $a = \infty$, and vice versa, then, for the sake of simplicity, some results further will be formulated namely for $a = \infty$.

Theorem 1

Let $L = \bigcup_{j=1}^{n} P_j(\overline{R})$, where $n \in N$, $P_j(z) = c_m^{(j)} z^m + c_{m-1}^{(j)} z^{m-1} + ... + c_0^{(j)}$, $m \in N$, $c_m^{(j)} \neq 0$, $j = 1,2,..., n$ and let $c_k^{(j)} = p_k(e_j)$, where the polynomials $p_k(z) \in \mathbf{C}[z] \setminus \mathbf{C}$, $k = 0,1,..., m$, and the numbers $e_1, e_2,..., e_n$ are all complex n-th roots of 1 (unity), the polynomial $p_m(z)$ has no complex unit module roots, then for the function

$$F(z) = p_m\left(\frac{\overline{z}}{z}\right)\left(i\overline{z}^n - iz^n\right)^m + p_{m-1}\left(\frac{\overline{z}}{z}\right)\left(i\overline{z}^n - iz^n\right)^{m-1} + ... + p_0\left(\frac{\overline{z}}{z}\right)$$

the congruence $C(F(z), \infty) = L$ is correct.

The deduction of the theorem 1 is in [6].

Samples

1. As $C(\overline{z} - z, \infty) = i\overline{R}$, then $C\left(\overline{z}^2 - z^2 + \frac{\overline{z}}{z}, \infty\right) = \left(i\overline{R} + 1\right) \mathbf{U} \left(i\overline{R} - 1\right)$ is a union of a pair of parallels;

$C\left(i\overline{z}^3 - iz^3 + \frac{\overline{z}}{z}, \infty\right) = \overline{R} \mathbf{U} \left(e_1 + \overline{R}\right) \mathbf{U} \left(e_2 + \overline{R}\right)$ is a union of three parallels

$\left(e_1 = e^{i\frac{2p}{3}},\ e_2 = e^{i\frac{4p}{3}}\right)$;

$C\left(i\frac{\overline{z}}{z}\left(\overline{z}^3 - z^3\right), \infty\right) = \overline{R} \mathbf{U} e_1\overline{R} \mathbf{U} e_2\overline{R}$ is a union of tree concurrent in the 0 point lines.

2. As $C\left(-\left(\overline{z} - z\right)^2, \infty\right) = \overline{R}_+$, то $C\left(\left(-\left(\overline{z}^2 - z^2\right)^2 + 1\right)\frac{\overline{z}}{z}, \infty\right) = (-\infty, -1] \mathbf{U} [1; +\infty)$;

$C\left(-\frac{\overline{z}}{z}\left(\overline{z}^3 - z^3\right)^2, \infty\right) = \overline{R}_+ \mathbf{U} e_1\overline{R}_+ \mathbf{U} e_2\overline{R}_+$ is a union of three half-lines, all of them centering in 0 point and making angles of 120^0.

3. As $C\left((\bar{z}+z+i)^2+1,\infty\right)=\left\{t^2+2ti \mid t\in \bar{R}\right\}$ is a parabola of the second order, then every of the sets, $C\left(\frac{\bar{z}}{z}\left((\bar{z}^2+z^2+i)^2+1\right),\infty\right)$ and $C\left((\bar{z}^2+z^2+i)^2+1+\frac{\bar{z}}{z},\infty\right)$, is a union of two parabolas.

In the conclusion of the article let us show some simple upper estimate of the number of polynomial lines, making up $C(F(z),a)$, where $a\in \overline{\mathbf{C}}$ is isolated singular l-point of the p.a. function $F(z)$.

Theorem 2

For any p.a. function $F(z)$ of the proximate poly-analyticity order $n\in N,\ n\geq 2$, and for its every isolated singular l-point $a\in \overline{\mathbf{C}}$ the set of all the elements from $C(F(z),a) \cap \mathbf{C}$ can be represented in the form of a union of finite number of nontrivial polynomial lines, the quantity of which $l\in N$ satisfies the following conditions:

a). $l\leq 4(n-1)$;

б). $l\leq (n-1)!$ (with $n=2$ and with $n=3$ this estimate is exact).

The deduction of the theorem 2 is in [6].

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CHRACTERISTIC PROPERTIES OF SEQUENCES GENERATING FINITE ELEMENTS OF POLYANALYTIC FUNCTIONS' CLUSTER SETS IN THEIR ISOLATED SINGULAR L-POINTS

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1. For every poly-analytic function (p.a. function) [1-4]